

Housing Quality Differentials  
In Urban Areas

by

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## ABSTRACT

This paper applies an equilibrium quality theory for differentiated products to estimate the willingness to pay for improvements in the air quality of Chicago, Cleveland, Dallas, Houston, and Indianapolis. The empirical results show (i) that the structural approach and the standard non-structural approach give very different benefit figures even for small improvements in air quality, and (ii) that a uniform improvement in air quality implies significant distributional effects.

## I. Introduction and Summary.

Houthakker (1952) assumed that the characteristics of commodities provide utility to individuals and introduced a new approach to the problem of quality variation and to the theory of consumer behavior. This new approach to the theory of individual choices helps to explain a number of phenomena that the traditional economic theory cannot easily explain.

In recent years, several economists have adopted the new approach to the theory of individual choices and have extended Houthakker's analysis to study consumer behavior. For example, Becker (1965), Lancaster (1966), and Muth (1966) assumed that commodities traded in the market do not possess final consumption attributes and that consumers are also producers, that is, the consumers are assumed to use the commodities purchased in the market as inputs into a self-production function for ultimate characteristics. Becker, Lancaster, and Muth did not study producer behavior and the properties of market equilibrium. Rosen (1976) studies both consumer and producer behavior and the properties of market equilibrium. Unlike the previous work in the area, Rosen assumes that consumers are not producers and that all the commodities with their ultimate characteristics are readily available and traded in the market.

Epple (1987) demonstrates that most of the work that uses the hedonic approach is unsatisfying because the estimation methods do not yield consistent estimates. Bartik (1987) and Palmquist (1984) are two exceptions. With few exceptions, the hedonic approach has not been analyzed thoroughly,

complete hedonic equilibrium models have not been estimated and none of the previous application contains a structural analysis. Depending on the structure of the economy and on the questions that we want to address, we do not always need to compute closed-form solutions and make a structural analysis. For example, the standard approach can estimate the price equation and the parameters of the demand for product characteristics. However, structural analysis is needed to compute the effects of changes in exogenous parameters. Changes in exogenous parameters change the equilibrium price distribution and non-structural approaches cannot take account of such changes.

Tinbergen (1959) provided the earliest contribution to the formulation and solution of hedonic equilibrium models and Epple (1984) generalizes Tinbergen's model to treat a commodity with an arbitrary number of attributes and an endogenous supply for product characteristics<sup>1</sup>. However, these models have not been used for empirical work because they have several restrictive features. Namely, 1) the cross partial derivative of the utility function with respect to the quality characteristics of the differentiated good and the numeraire good is zero, 2) the marginal utility with respect to the numeraire good is constant (hence the income elasticity of demand for the product is zero), 3) the variance-covariance matrices of the exogenously given distributions have to be diagonal or satisfy other restrictions, 4) the number of consumer characteristics equals the number of product characteristics, and 5) the price equation parameters are not unique.

This paper presents an equilibrium model for the differentiated good housing. This model assumes that a linear function maps physical characteristics into a scalar quality index and that economic agents care only about the quality of the differentiated good that they purchase. While this is a strong assumption that is not present in the Tinbergen-Epple's formulation, this quality index technology allows me to impose weaker a priori restrictions in other respects. The result is a housing model with a closed-form solution that does not have the five restrictive features enumerated above. In addition to a vector of consumer characteristics, this model uses consumer income to distinguish individuals.

This model uses an analytically consistent description of the hedonic property value model i) to estimate the utility and the equilibrium price and demand functions and ii) to investigate how far one can go with closed-form solutions and how well the resulting model fits the data. The empirical results are used for an analysis of the housing market and an estimation of the willingness to pay for air quality improvements in Chicago (Illinois), Cleveland (Ohio), Dallas (Texas), Houston (Texas), and Indianapolis (Indiana).

There are many different methods one could use to characterize the hedonic equilibrium. One would be to empirically approximate the features of the price function using fitting criteria to derive it. This method provides more flexibility in letting the data determine the price equation and willingness to pay functional forms at the cost of not being able to test whether the assumed functional forms are consistent among themselves and the

underlying economic structure. Another method, that is followed by this paper, makes prior assumptions about the characteristics of the economic agents interacting to form the equilibrium, uses that to derive the form of the equilibrium hedonic function, and then estimates only that. Imposing these prior restrictions helps through the additional theoretical information that is essential in the derivation of the willingness to pay results.

There is an inevitable trade off associated with a simplification required to derive an analytically consistent form for a hedonic price function. In this paper, all of the analysis underlying the derivation of the equilibrium price equation and the method for estimating the parameters of the equilibrium price equation, of the utility function, and of the associated willingness to pay function rest on the assumptions 1) that the utility function is quadratic, 2) that the vector of product characteristics associated with the housing and the vector of family size and income follow multivariate normal distributions in each city, and 3) the linearity of the housing quality variable entering the utility function. The latter is what allows me to reduce a not-diagonal variance-covariance structure into a simple variance and apply the basic argument developed in Epple (1984). A quadratic utility function and normality distributional assumptions also characterize Epple's (1984) analysis.

Quigley (1982) uses an alternative strategy to derive a method for estimating individual preferences and the price of a differentiated product. This method, as well as the model presented in Polinsky and Rubinfeld

(1977), cannot be used for a general equilibrium analysis because they do not provide consistent estimates of the equilibrium price equation.

Section II introduces the theoretical housing model that is used to illustrate the kind of analysis that the structural approach can perform. This model assumes that the income and the supply distributions are exogenous and that consumers use the services of only one unit of the differentiated good. However, the same basic model can be extended to relax these assumptions (see Giannias (1987)). An application of the model is discussed in Section III. Concluding remarks are presented in Section IV.

## II. The Economic Model.

The differentiated good housing can be accurately described by a  $(1 \times m)$  vector of objectively measured characteristics,  $v$ . Individuals consume one unit of housing and the numeraire good,  $x$ . It is assumed that they care only about the quality of the differentiated good housing,  $h$ , and that the housing market is competitive. The housing quality,  $h$ , is a scalar and a linear function of the vector of physical characteristics,  $v$ , that is,

$$h = \epsilon v' \tag{1}$$

where  $\epsilon$  is a  $(1 \times m)$  vector of parameters and  $v'$  is the transpose of  $v$ . (Hereafter, a prime "'" will always denote the transpose of a vector or matrix). Equation (1) is a key assumption of the model. The equation is less restrictive than might at first appear since an element of  $v$  can be an arbitrary function of measured product characteristics. The supply for housing characteristics is exogenously given and the vector  $v$  is assumed to

follow an exogenously given normal distribution with a mean  $\bar{v}$  and a variance-covariance matrix  $\Sigma_v$ . The latter assumption and (1) imply that the aggregate supply for housing quality follows a normal distribution with a mean  $\bar{h}$  and a variance  $\sigma^2$ . Let it be

$$g(h) = N(\bar{h}, \sigma^2) \quad (2)$$

where  $\bar{h} = \epsilon \bar{v}'$ , and  $\sigma^2 = \epsilon \Sigma_v \epsilon'$ .

The model lets consumers have different preferences and income. Each consumer can be described by a vector  $z$ , where  $z = [a \ I]$  is a  $[1 \times (n+1)]$  vector,  $I$  is the consumer income, and  $a$  is a  $(1 \times n)$  vector of utility parameters that specifies the type of a consumer.  $z$  is assumed to follow a multi-normal distribution with a mean  $\bar{z}$  and a variance-covariance matrix  $\Sigma_z$ .

$U(h, x; a)$  is the utility that an  $a$ -type consumer obtains from  $x$  and from the services of an  $h$ -quality house. The utility function is assumed to be a quadratic of the following form:

$$U(h, x; a) = \delta + (\zeta_0 + \zeta_1 a') h + 0.5 \xi h^2 + x h \quad (3)$$

where  $\delta$ ,  $\zeta_0$ , and  $\xi$  are utility parameters (scalars), and  $\zeta_1$  is a  $(1 \times n)$  vector of utility parameters.

An  $a$ -type consumer with an  $I$  annual income solves the following optimization problem:

$$\max U(h, x; a) \quad (4)$$

with respect to  $h, x$

subject to  $I = 12 P(h) + 365 x$  and

$$P(h) = \pi_0 + \pi_1 h$$



where  $\pi_i$  is a parameter,  $i = 1, 2$ ,  $P(h)$  is the equilibrium housing price equation (it is a function of housing quality and equal to gross monthly rent including utilities), 12 is the number of months in a year, and 365 is the number of days in a year.

The optimum decisions of housing sellers and consumers depend on the equilibrium housing price equation  $P(h)$  which is determined so that housing buyers and sellers are perfectly matched. In equilibrium, no one of the economic agents can improve his position and all of their optimum decisions are feasible.

Solving the utility maximization problem, it is obtained that the demand for  $h$  is given by the following equation:

$$h = (\zeta_0 - \pi_0 + \tau z') / (2 \pi_1 - \xi) \quad (5)$$

where  $\tau = [\zeta_1 \quad 1/365]$  is a  $[1 \times (n+1)]$  vector.

The normality of  $z$  and (5) imply that the aggregate demand for housing quality follows a normal distribution. Let it be  $f(h)$ . The condition for an equilibrium in the market described above is: aggregate demand for housing quality = aggregate supply for housing quality, that is,  $f(h)dh = g(h)dh$ .  $P(h) = \pi_0 + \pi_1 h$  is the equilibrium housing price equation<sup>2</sup> because it sets the mean and variance of the aggregate demand for  $h$  equal the mean and the variance of the aggregate supply for  $h$  respectively, where<sup>3</sup>

$$\pi_1 = (365/24) [\xi + (\tau \Sigma_z \tau' / \sigma^2)^{0.5}] , \text{ and} \quad (6)$$

$$\pi_0 = (365/12) [\zeta_0 + \tau \bar{z}' - (2 \pi_1 - \xi) \bar{h}] \quad (7)$$

The housing price equation given above is an equilibrium price relationship that is determined by the distribution of consumer tastes and income, and by the distribution of housing characteristics. In Section III, the model described above is used for a study of the residential housing market and an illustration of the method that can be applied for estimating the willingness to pay for changes in exogenous parameters, namely, the mean of the air quality distribution.

### III. An application.

The model that is presented in the previous section can be used for a study of the residential housing market of Chicago (Illinois), Cleveland (Ohio), Dallas (Texas), Houston (Texas), and Indianapolis (Indiana). The empirical results will be used to investigate the willingness to pay for air quality improvements in these cities.

#### III.A. Additional Assumptions and Definitions.

It is assumed that there is no migration among cities and that the differentiated product residential housing can be described by a vector of characteristics  $v$ , where  $v = [v_1 \ v_2 \ v_3]$ ,  $v_1$  is the size of the housing unit (number of rooms),  $v_2$  is an air quality index, and  $v_3$  is the travel time to work (measured in minutes).  $v_2$  equals the inverse of the air pollution variable total suspended particulate matter. The housing quality equation is given in (1), where  $\epsilon = [\epsilon_0 \ \epsilon_1 \ \epsilon_2]$ . The parameters of the quality equation should be indexed by  $j$ , where  $j =$  Chicago, Cleveland, Dallas,

Houston, Indianapolis. However, the subscript  $j$  has been dropped to simplify the notation.

Consumer preferences are described by the utility function given in equation (3). The parameter  $a$  is defined to be the number of persons in a family (a scalar). The parameters of the utility function and the parameters of the distributions of the vector  $[a \ I]$  and of the vector of housing characteristics,  $v$ , should also be indexed by  $j$ , where  $j =$  Chicago, Cleveland, Dallas, Houston, Indianapolis. As above, the subscript  $j$  has been dropped to simplify the notation. A consumer solves the optimization problem given in (4).

### III.B. The Econometric Model.

To obtain the equilibrium demand for housing quality, I substitute (6) and (7) into (5). Assuming an additive error term on the price equation and on the equilibrium demand for housing quality, I obtain:

$$P = c + \beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3 + u_1 \quad (8)$$

$$h = \gamma - \epsilon_3 a - \epsilon_4 I + u_2 \quad (9)$$

where  $c = (365/12) [\zeta_0 + \zeta_1 \bar{a} + (\bar{I}/365) - A \epsilon \bar{v}'] \quad (10)$

$$\beta_{i+1} = (365/24) (\xi + A) \epsilon_i, \text{ for } i = 0, 1, 2 \quad (11)$$

$$\gamma = \bar{v}_1 + \epsilon_1 \bar{v}_2 + \epsilon_2 \bar{v}_3 + \epsilon_3 \bar{a} + \epsilon_4 \bar{I} \quad (12)$$

$$\epsilon_3 = - \zeta_1/A \quad (13)$$

$$\epsilon_4 = - 1/(365 A) \quad (14)$$

$$A = ( \tau \Sigma_2 \tau' / \sigma^2 )^{0.5},$$

a " $\bar{\quad}$ " over a variable denotes the mean of the variable, and  $u_1$  and  $u_2$  are

the econometric errors of the first and second equation respectively. The complete model consists of equations (1), (8), and (9).

Among the parameters of the equilibrium price equation, the parameters of the equilibrium demand for housing quality, and the parameters of the quality index equation,  $\beta_2$ ,  $\epsilon_1$ , and  $\gamma$  are assumed to be the only ones that can be different across cities. Moreover, I make the fixed effect assumption that these three parameters satisfy the following:

$$\beta_2 = \beta_{20} + \sum_{i=1}^4 \beta_{2i} d_i \quad (15)$$

$$\epsilon_1 = \epsilon_{10} + \sum_{i=1}^4 \epsilon_{1i} d_i \quad (16)$$

$$\gamma = \gamma_{10} + \sum_{i=1}^4 \gamma_{1i} d_i \quad (17)$$

where  $\beta_{2j}$ ,  $\epsilon_{1j}$ , and  $\gamma_{1j}$  are exogenously given parameters (they are some of the parameters that I want to estimate) for  $j = 0, 1, 2, 3, 4$ , and  $d_1 = 1$  for Chicago and 0 else,  $d_2 = 1$  for Cleveland and 0 else,  $d_3 = 1$  for Dallas and 0 else, and  $d_4 = 1$  for Indianapolis and 0 else.

Differences in exogenous factors, for example, humidity, temperature, and rainfall, can make the parameter of the quality index equation  $\epsilon_1$  be different across cities. For the residential housing market, I assume that the quality of housing is a latent variable. Without loss of generality, the quality of housing can be normalized by setting  $\epsilon_0$  equal to 1.

The econometric errors are assumed to satisfy: (A1)  $u_1$  and  $u_2$  are uncorrelated, (A2)  $a$  and  $I$  are uncorrelated to  $u_1$  and  $u_2$ , and (A3)  $v_1$ ,  $v_2$ , and  $v_3$  are uncorrelated to  $u_1$ . These assumptions may be motivated, for example, by thinking of  $u_1$  as a measurement error in price and  $u_2$  as

unmeasured buyer characteristics that are uncorrelated with measured buyer characteristics.

### III.C. Estimation of the Reduced Form Equations.

To estimate the complete model, I apply a four step estimation procedure. This estimation method yields consistent parameter estimates and uses the restrictions that are implied by the structure of the model, namely,

$$\epsilon_1 = \beta_2/\beta_1, \text{ and} \tag{18}$$

$$\epsilon_2 = \beta_3/\beta_1 \tag{19}$$

I estimate the model for Chicago (Illinois), Cleveland (Ohio), Dallas (Texas), Houston (Texas), and Indianapolis (Indiana) using 1980 census tract data on gross rental prices, number of rooms, travel time to work, size of the family, and income, and 1979 SAROAD based data on air pollution. To obtain data concerning the annual arithmetic mean of total suspended particulate (measured in microgram per cubic meter) all the monitoring stations in these five cities (given their addresses) were located according to census tract. The readings for these census tracts were used to represent pollution readings in adjacent census tracts since most cities contain a limited number of monitoring stations. If a census tract was adjacent to more than one census tract containing a monitoring station, then the average of the readings was used. These readings were then inverted, such that the figures reflect air quality instead of air pollution.

Unlike other work, e.g., Harrison and Rubinfeld (1978), the model has nice aggregation properties that allow mean values of census tract data to be used. If micro data on individual consumers is not available, the use of census tract data can be justified since 1) the price equation is linear in product characteristics, and 2) the equilibrium demand for product quality is linear in consumer income and family size. The estimation method follows.

STEP 1: I estimate the price equation by ordinary least squares (which is appropriate under assumption A3). The parameter estimates are given in Table 1. They imply that the rental price equation for each city is given by the following equation:

$$P = 85.06 + 19.49 v_1 + \beta_2 v_2 - 4.56 v_3 \quad (20)$$

where the value of the parameter  $\beta_2$  for each city is given in Table 2.

STEP 2: Given (18) and (19) and the results of the previous step, I can obtain estimates for  $\epsilon_1$  and  $\epsilon_2$ . This and the normalization  $\epsilon_0 = 1$  enable me to obtain that the housing quality index equation for each city is given by the following equation:

$$h = v_1 + \epsilon_1 v_2 - 0.23 v_3 \quad (21)$$

where the value of the parameter  $\epsilon_1$  for each city is given in Table 2.

STEP 3: I use the above specified housing quality equations to construct an estimated series for the housing quality for each census tract of my data set.

STEP 4: I use the housing quality indices that I obtained in step 3 to estimate equation (9). Ordinary least squares is appropriate under assumptions A1-A3. Deviations between the actual housing quality and its estimate (estimated from equation (21)) are measurement errors in the

dependent variable in equation (9) and hence do not affect the consistency of ordinary least squares. The parameter estimates are given in Table 3. They imply that the equilibrium demand equation for each city is given by the following equation:

$$h = \gamma + 0.138 a + 0.000139 I \quad (22)$$

where the value of the parameter  $\gamma$  for each city is given in Table 2.

To see if the model makes a significant contribution to explaining the data, I tested the hypothesis that all the parameters of equation (8) are zero, that is,  $\beta_1 = \beta_{20} = \beta_{21} = \beta_{22} = \beta_{23} = \beta_{24} = \beta_3 = 0$ . An F-test rejects that hypothesis at the 1% significance level. A similar F-test rejects the hypothesis (at the 1% significance level) that all the parameters of equation (9) equal zero.

The t-statistics (see Tables 1 and 3) show that the size of a house (number of rooms) and the travel time to work variable (which are expected to be the main determinants of the rent), as well as the income (which is expected to be the main determinant of the equilibrium demand for housing quality) are significant at the 1% significance level. Moreover, all coefficients have the anticipated signs in both equations (8) and (9) and the qualitative properties of the model are as one would intuitively expect<sup>4</sup>.

Various alternative hedonic specifications were estimated pooling the same data. The equation given in Table 1 did not change significantly under the alternative specifications. To investigate for possible non-linearities,

the following equation is estimated:  $P = b_0 + \sum_i b_i v_i + \sum_{ij} b_{ij} v_i v_j$ . An F-test provides evidence in favor of the null hypothesis that  $b_{ij} = 0$  for all  $i = 1, 2, 3$  and  $j = 1, 2, 3$ .

Blomquist and al (1985) and (1988) have estimated (among others) a housing hedonic equation that includes housing structural characteristics, urban characteristics, climatic conditions, and environmental variables. Their analysis includes the five cities that are considered in this paper. Their (1985) paper assumes a log-log functional form for the hedonic housing equation and in their (1988) paper they have applied the Box-Cox procedure, see Box and Cox (1962). In both papers, they have used 1980 Census earnings and housing micro data for individuals. However, their air pollution variable is more aggregate than the one that it has been used in this paper (the unit observation for their air pollution variable is the county or SMSA; the same is true for other variables as well). Table 4 gives the elasticities of rental prices with respect to number of rooms and air quality,  $e(P, v_1)$  and  $e(P, v_2)$  respectively, that are implied by the empirical results of this paper and the work of Blomquist and al<sup>5</sup>. Tables 4 shows significantly different figures for the elasticities of rents with respect to air quality<sup>6</sup>.

#### IV.D. Structural Analysis.

The parameter estimates that I obtained in the previous section allow me to analyze the structure of the housing market of Chicago, Cleveland, Dallas, Houston, and Indianapolis, and to specify how that structure depends



on the mean of the air quality distribution of each city. The latter enables me to address interesting questions that a non-structural approach cannot.

Given the parameter estimates obtained in Section IV.C. and equations (10)-(19), I can compute the parameters of the utility function and the equilibrium demand for the numeraire good<sup>7</sup>. For each city, the equilibrium demand for the numeraire good and the utility functions<sup>8</sup> are respectively given by the following equations:

$$x = d - 0.088 a + 0.00265 I \quad , \text{ and}$$

$$U(h,x;a) = \delta + (\zeta_0 + 2.72 a) h - 9.21 h^2 + x h \quad (23)$$

where the values of the parameters  $d$  and  $\zeta_0$  for each city are given in Table 2. We can now see that 1) the rent is positively related to the quality of a house<sup>9</sup>, 2) the equilibrium demand for housing quality is positively related to the size of the family and income (see equation (22)), 3) the housing quality is positively related to air quality and negatively to travel time to work (see equation (21)), and 4) the marginal utility with respect to housing quality is positively related to the size of a family (see equation (23)). These qualitative properties are as one would intuitively expect<sup>10</sup>.

The preceding results can be used in a structural analysis of the value of a change in air quality. To illustrate that, I first repeat the above calculations treating mean air quality,  $\bar{v}_2$ , as a variable rather than fixing it at its sample mean of (see Table 2 for the mean air quality of each city). The results follow.

The parameters  $A$ ,  $\zeta_0$ ,  $\zeta_1$ ,  $\epsilon_1$ ,  $\epsilon_2$ , and  $\xi$  do not change because they do

not depend on the mean air quality. The housing quality index equations and the utility functions are given in (21) and (23) respectively. The equilibrium rental price equations, the equilibrium demand for housing quality equations, and the equilibrium demand for the numeraire good equations are functions of the mean air quality. They are respectively equal to:

$$P = r + k \bar{v}_2 + 19.49 h \quad (24)$$

$$h = s + m \bar{v}_2 + 0.138 a + 0.000139 I \quad (25)$$

$$x = y + n \bar{v}_2 - 0.088 a + 0.00265 I \quad (26)$$

where the values of the parameters  $r$ ,  $k$ ,  $s$ ,  $m$ ,  $y$ , and  $n$  for each city are given in Table 2.

To illustrate how to perform a general equilibrium analysis that is accommodated by the model, these results are used to compute the willingness to pay for an improvement in air quality. The purpose of this is not to determine the precise dollar figure of the willingness to pay for an improvement in air quality. Rather, it is to contrast this method to the previous (partial equilibrium) common practice for computing benefits.

A consumer's willingness to pay for a  $y\%$  improvement in the mean air quality of a city,  $W$ , is defined to be the solution to the following equation:

$$V(a, I, \bar{v}_2) = V( a, I + W, \bar{v}_2 + \bar{v}_2 (y/100) ) \quad (27)$$

where  $V(a, I, \bar{v}_2)$  is the equilibrium indirect utility function of an  $[a I]$ -consumer given the mean air quality of a city,  $\bar{v}_2$ . That is, the consumer's benefit from a  $y\%$  change in the mean air quality is the part of his income that he is willing to give up so that the utility after the  $y\%$  change,

taking account of equilibrium price adjustments, equals the utility before the y% change.

The willingness to pay of the mean household of each city for a y% air quality improvement<sup>11</sup>,  $W(y\%)$ , is computed for  $y = 1, 2.5, 5, 7.5, 10, 12.5,$  and 15 using numerical procedures that are available in the TKSolver computer package. The results are summarized in Figure 1. They indicate that the willingness to pay for an air quality improvement in Cleveland and Indianapolis is significantly lower than the willingness to pay for the same air quality improvement in Chicago, Dallas, and Houston, for all y% air quality improvements.

A uniform air quality improvement i) shifts the price equation downwards (see equation (24)) and ii) increases the quality of all houses (see equation(21)). The empirical results imply that the net effect is a decrease in the rent of all houses. Table 5 gives the willingness to pay for a 1% air quality improvement,  $W(1\%)$ , as well as the annual decrease in rent revenues of the mean house of each city that is implied by that 1% air quality improvement,  $\Delta R(1\%)$ . The third column of Table 5 gives the net social benefit per household<sup>12</sup>,  $NSB(1\%)$ , and the fourth column gives the willingness to pay that a standard non-structural approach would yield given the price equation in Table 1. That is, the figures of the fourth column are computed in the following way<sup>13</sup>:  $W = 12 \beta_2 (DV)$ , where  $\beta_2$  is the coefficient of the air quality variable of equation (20) (given in Table 2) and DV is the change in the mean air quality,  $DV = 1\%$ . The benefit figures of the fourth column are 97% below the benefit figures based on the structural

model (first column of Table 5). This difference arises only because of differences in method of calculation, since the same price equation parameters were used for both calculations. Consequently, the method of computing benefits matters a lot and a non-structural approach will not necessarily give a good approximation to the benefit figure that is implied by a structural approach even for small changes in air quality.

For a small 1% air quality improvement, it is also obtained that our results are consistent with the Pines and Wines (1976) marginal result (see the third and fourth columns of Table 5). Pines and Weiss have shown that the marginal price paid for an amenity improvement that is marginal at all locations (even if large in the sense that many locations are improved), when summed over all improved locations will give an accurate measure of net social benefits. Table 5 also implies that the most significant effect of an air quality improvement is an enormous distributional effect since the figures of the third column are on average 96% below the figures of the first column.

#### IV. Conclusions.

This paper presents a model for the differentiated good housing that makes prior assumptions about the characteristics of the economic agents interacting to form the equilibrium and uses that to derive and estimate the equilibrium price equation, the housing quality index equation, the equilibrium demand for housing quality, and the parameters of the utility function. These prior restrictions are essential in the estimation of the

structural model and the derivation of the willingness to pay results. The empirical results indicate (i) that there is a significant distributional effect associated with a uniform improvement in air quality that cannot be identified by a non-structural approach, and (ii) that the willingness to pay for an air quality improvement in Cleveland and Indianapolis is a lot less than the willingness to pay for the same air quality improvement in Chicago, Dallas, and Houston. It is also an interesting result that the two methods that were used to compute benefits give very different benefit figures for a small 1% air quality improvement. The latter indicates that the non-structural approach can even miscalculate benefits of small changes in the air quality distribution.

TABLE 1  
THE RENTAL PRICE EQUATION

VARIABLE -----	COEFFICIENT -----	STANDARD ERROR -----	T-STATISTIC -----
v <sub>1</sub>	19.48912	6.027161	3.233549
v <sub>2</sub>	12785.74	1767.226	7.234924
v <sub>3</sub>	-4.557341	9.261684	-4.920640
d <sub>1</sub> v <sub>2</sub>	1041.140	1177.005	0.8845673
d <sub>2</sub> v <sub>2</sub>	-6897.647	1205.995	-5.719468
d <sub>3</sub> v <sub>2</sub>	-3670.402	896.7612	-4.092954
d <sub>4</sub> v <sub>2</sub>	-8348.988	1192.448	-7.001555
INTERCEPT	85.05893	36.63616	2.321720

N = 152

R<sup>2</sup> = .51

NOTE: N is the number of observations.

TABLE 2

PARAMETER VALUES AND STATISTICS FOR EACH CITY

	CHICAGO -----	CLEVELAND -----	DALLAS -----	HOUSTON -----	INDIANAPOLIS -----
$\beta_2$	13826.88	5888.09	9115.34	12785.74	4436.75
$\epsilon_1$	709.43	302.11	467.69	656.01	227.64
$\gamma$	-5.53	-3.11	-3.95	-4.51	-2.41
d	-0.6088	-0.8009	-0.2652	-0.0927	-1.2599
$\zeta_0$	-102.26	-58.50	-75.06	-86.10	-44.70
r	5231.34	2004.92	4767.51	5630.39	1845.56
k	-425312.15	-181118.72	-280386	-393286.2	-136473.03
s	13.91	-6.31	-11.76	-13.76	-5.35
m	709.43	302.11	467.69	656.01	227.64
y	-163.09	-61.87	-149.21	-176.3	-57.26
n	13528.825	5761.24	8918.85	12510.12	4341.09
$\bar{a}$	1.95	1.84	2.19	2.5	1.85
$\bar{I}$	16928	10784	15795	15954	13889
$\bar{v}_2$	0.0121	0.0106	0.0167	0.0141	0.0129

TABLE 3

## THE DEMAND FOR HOUSING QUALITY EQUATION

VARIABLE -----	COEFFICIENT -----	STANDARD ERROR -----	T-STATISTIC -----
I	0.0001393595	0.0000202607	6.878315
a	0.1379081	0.133066	1.036389
d <sub>1</sub>	-0.8252417	0.2822666	-2.923625
d <sub>2</sub>	1.401357	0.2914920	4.807532
d <sub>3</sub>	0.558272	0.2640562	2.114217
d <sub>4</sub>	2.102350	0.3154746	6.664085
INTERCEPT	-4.509668	0.5332514	-8.456927

N = 152

R<sup>2</sup> = .41



TABLE 4  
PRICE ELASTICITIES WITH RESPECT TO AIR QUALITY  
AND NUMBER OF ROOMS

	$e(P, v_1)$	$e(P, v_2)$	Blomquist and al		
			(1985)		(1988)
			$e(P, v_1)$	$e(P, v_2)$	$e(P, v_2)$
CHICAGO	0.42	0.74	0.30	0.121	0.23
CLEVELAND	0.70	0.50	0.30	0.121	0.40
DALLAS	0.35	0.73	0.30	0.121	0.13
HOUSTON	0.35	0.79	0.30	0.121	0.15
INDIANAPOLIS	0.62	0.45	0.30	0.121	0.43

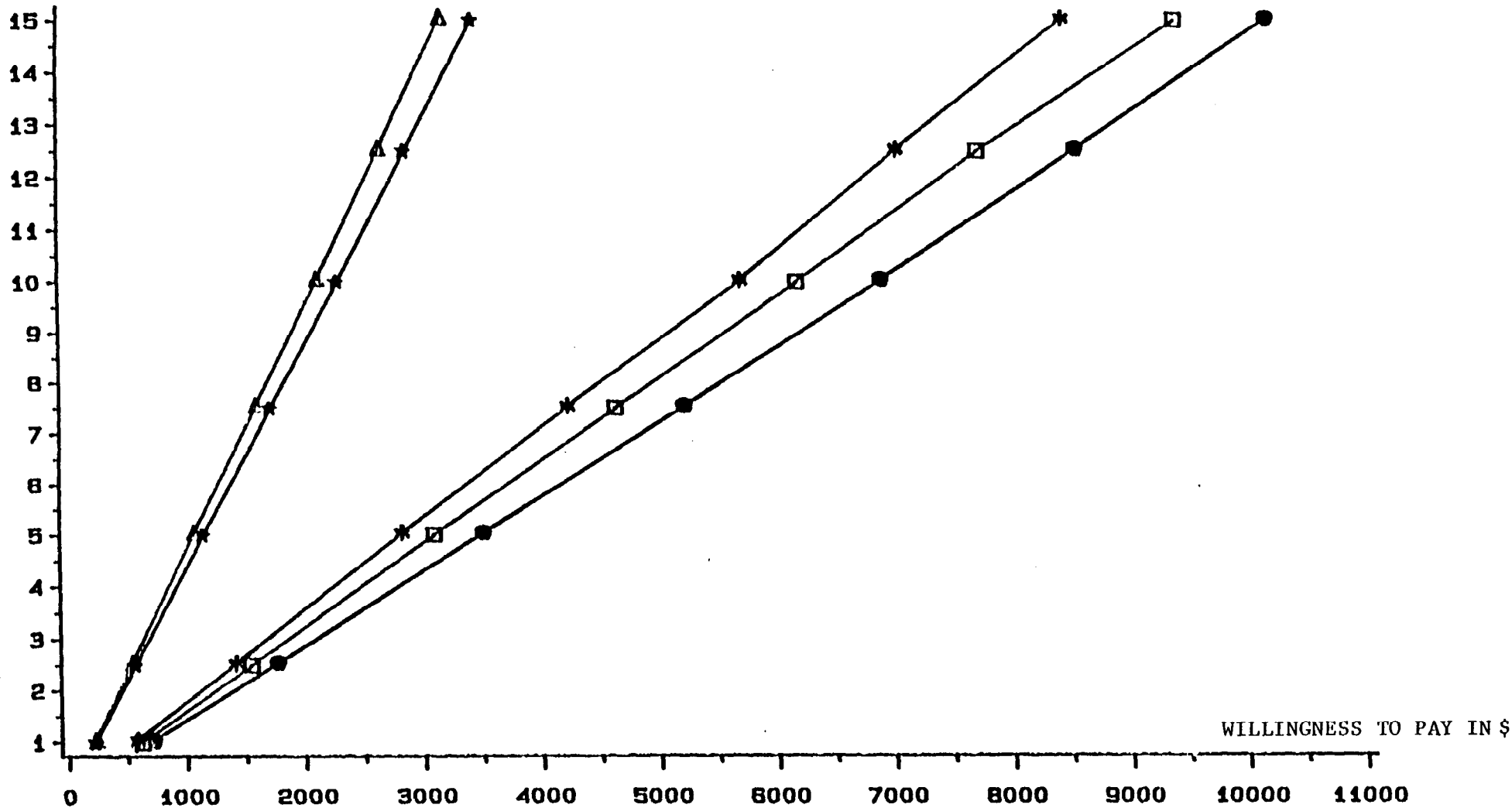
TABLE 5

THE WILLINGNESS TO PAY OF THE MEAN HOUSEHOLD, THE CHANGE IN THE RENT REVENUES OF THE MEAN HOUSE, AND THE NET SOCIAL BENEFIT PER HOUSEHOLD THE ASSOCIATED WITH AN 1% AIR QUALITY IMPROVEMENT IN THE MEAN AIR QUALITY OF EACH CITY

	W(1%)	$\Delta R(1\%)$	NSB(1%)	$W=12 \beta_2 1\%$
CHICAGO	615.6	597.36	18.24	20.1
CLEVELAND	229.7	222.96	6.74	7.5
DALLAS	562.8	543.60	19.2	18.3
HOUSTON	709.6	683.92	25.68	21.6
INDIANAPOLIS	212.2	204.48	7.72	6.9

# WILLINGNESS TO PAY VS. IMPROVEMENT

% AIR QUALITY  
IMPROVEMENT



CITY

●-●-● HOUSTON

△-△-△ INDIANAPOLIS

□-□-□ CHICAGO

\*-\*-\* DALLAS

\*-\*-\* CLEVELAND

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## ENDNOTES

1. McConnell and Phipps have also developed a variety of extensions to the model in unpublished work.
2. The general strategy of the proof was introduced by Tinbergen (1959) and extended by Epple (1984).
3. There are two solutions that satisfy the equilibrium condition. The one of them is rejected because it does not satisfy the second order condition for utility maximization.
4. For the residential housing market, I expect the parameters  $\epsilon_1$ ,  $\epsilon_2$ , and  $\zeta_1$  to satisfy:  $\epsilon_1 > 0$ ,  $\epsilon_2 < 0$ , and  $\zeta_1 > 0$ . That is, I expect 1) the housing quality to increase as the air quality increases, 2) the housing quality to decrease as the travel time to work increases, and 3) the utility that is obtained from each additional unit of housing quality to increase as the size of a family increases. The parameter estimates obtained in this section show that the two of the above inequalities are satisfied. In the next section, it is shown that the third of them is also satisfied.
5. The information needed to compute the elasticity of rents with respect to the number of rooms is not reported in Blomquist and al (1988).
6. The elasticity of rents with respect to air quality predicted by Blomquist and al (1985) and (1988) is 80% and 51% respectively below the

elasticity figures predicted by the empirical results of this paper.

7. The equilibrium demand of the numeraire good is obtained from the budget constraint after substituting out the equilibrium demand for housing quality.

8. The intercept of the marginal utility of the housing quality can be different across cities because of differences in exogenous parameters, for example humidity, temperatures, rainfall, or other city specific characteristics.

9. To see this note that (18), (19), (20), and (21) imply the following equation:  $P = 85.06 + 19.49 h$ .

10. Moreover, the model predicts the following: 1) "Indianapolis, Cleveland, Dallas, Houston, Chicago" is the order in which a household (given a family size and income) would order the five cities (from most to least desirable) according to the housing quality that he can enjoy (buy in equilibrium) in each city, see equation (22), 2) the housing quality is more sensitive to changes in air quality in Chicago and less in Houston, Dallas, Cleveland, and Indianapolis (the order is from most to least sensitive); more sensitive in the sense that a unit change in air quality changes more the housing quality in Chicago than in other cities, see equation (21), and 3) identical houses (houses described by the same travel time to work, air quality, and number of rooms variables), have different rental prices in each city. Rents are more expensive in Chicago and less expensive in Houston, Dallas,



Cleveland, and Indianapolis (the order is from most to least expensive), see equation (20).

$$11. \text{NSB}(1\%) = \text{W}(1\%) - \Delta\text{R}(1\%) .$$

12. To obtain the equilibrium indirect utility function, I substitute the equilibrium demand functions for housing quality and numeraire good, equations (25) and (26) respectively, into the utility function, equation (23). Given this equilibrium indirect utility function, equation (27) can be obtained. The mean income, the mean number of persons in a family, and the mean air quality of each city (see Table 2) are substituted into (27) to solve for the willingness to pay of the mean household for several air quality improvements.

13. For example, this approach is used by Harrison and Rubinfeld (1978), p. 92, footnote 28. That is, for a price equation that is linear in air quality, they do not use their four step procedural model to compute benefit.