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Gibson's Paradox II

by

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ABSTRACT

The Gibson paradox, long observed by economists and named by John Maynard Keynes (1936), is a positive relationship between the interest rate and the price level. This paper explains the relationship by means of interest-rate, cost-push inflation. In the model, spending is driven in part by changes in the rate of interest, and the central bank sets the interest rate using a policy rule based on the levels of output and inflation. The model shows that the cost-push effect of inflation, long known as Gibson's paradox, intensifies destabilizing forces and can be involved in the generation of cycles.

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GIBSON’S PARADOX, MONETARY POLICY, AND THE EMERGENCE OF CYCLES

1. Introduction

The term “Gibson paradox” relates to observed positive correlation between the interest rate and the price level (Tooke 1838). One logical explanation, which has been propounded—but vigorously challenged—for ages is as follows. If interest rates are a cost of production and prices are based on costs, then interest rate rises would be passed along to consumers in the form of higher prices (Sraffa 1960; L. Taylor 1983; Garegnani 1983; Dutt 1990–91; Pivetti 2001; Barth and Ramey 2001). As Taylor (2004) points out, this notion, “that the price level (and, by extension, the inflation rate) depends positively on the interest rate [,] has a checkered history.” It should be noted that there is even some modern evidence of the Gibson paradox, known by econometricians as the “price puzzle”’ (Hanson 2005). If this highly unconventional view were correct, high interest rates would be exactly the wrong medicine for inflation.

This paper probes the implications of Gibson’s paradox, extending the model presented in Hannsgen (2004) with what I believe is a more satisfactory specification. First, a discussion of some of the theoretical issues involved precedes the substantive work of the paper.

2. The Role of the Gibson Paradox in Economic Models

As pointed out in Hannsgen (2004), if the cost-push channel of monetary transmission were operative, one might imagine that counterinflationary monetary policy would generate instability; a higher inflation level leads the authorities to increase interest rates.

This would have the effect of increasing inflation rather than containing it, forcing the authorities to again raise interest rates.

The cost-push effects of interest rates have been used in formal models in the past (L. Taylor 1983; Dutt 1990–91). In Taylor's model, for example, interest costs enter the pricing equation through working capital needs. In Dutt's (1990–91) model, interest costs affect the mark-up firms must charge. Despite the insightfulness of these models, they treat monetary policy in an exogenous sense, which may not capture dynamics that are policy driven. In Taylor's model, the money supply is exogenous. In Dutt's model, the interest rate is exogenous and the money supply endogenous. In the period since those articles, a relatively new approach to modeling policy has gained acceptance in some circles. Rather than taking monetary policy (the interest rate) as a constant, recent authors have made it a function of central bank target variables, such as inflation and the output gap (J. Taylor 1993). This paper seeks to marry this new way of modeling policy formation to a model in the spirit of L. Taylor and Dutt (Hannsgen 2004).

The characteristics of the model are then as follows. First, equilibrium prices are determined by costs, plus a markup. Interest rates are determined by the central bank. In this case, the central bank is assumed to target inflation and output. Turning to the dynamics of output, in addition to a standard interest-rate effect, a Minskyan effect of interest-rate changes on output is posited (Minsky 1986, esp. Ch. 9; Hannsgen 2005). Along with the use of a policy-setting rule, this effect is, as far as the author knows, a second novel addition to a Gibson-effect model in Hannsgen (2004) and this paper.

Though this new model is based on Hannsgen (2004), it adds several new features. First, the policy setting rule, which has incremental changes toward a level dictated by a Taylor rule, fits with the usual understanding of Taylor rules better than the old one, which was stated strictly in terms of the rate of change of the interest rate. Second, a real interest rate *level* argument has been added to the aggregate demand equation, making the model more general. Third, to abstract from extraneous issues about incorrect price expectations affecting the real wage, "myopic perfect foresight" has been added to the wage Phillips curve, meaning that agents know, and act upon, the current derivative of the price level. Fourth, a markup has been added to the price-setting process; this puts the model in the somewhat more natural environment of monopolistic price setting. Fifth, a clearer explanation of the adjustment process governing output has been added. The result of all these changes is a much more complex model, which reduces to a system of three nonlinear differential equations in three variables, rather than two equations in two variables. The proof of the dynamics is commensurately more involved.

The results of the model are as follows. Proposition 1 below states that if three conditions on the parameters are satisfied, a limit cycle exists locally at the (conjectured) equilibrium point: in inflation-output-interest rate space, there is a closed cyclical path outside of the equilibrium point. The economy can be attracted to the cycle from everywhere in the state space except one. Alternatively, a "corridor of stability" exists, within which the economy tends to move toward the center. Propositions 2 and 3 in the paper show that the sensitivity of the policy reaction function (Taylor rule) is an important determinant of the stability of the system— if it is too low *or* too high, the system will be unstable. The paper will now demonstrate how these results come about.

3. The Model

Prices are driven by costs in this model. It is assumed that there are two factors of production: labor and bank loans. The production technology dictates that firms must hire "a" units of labor for each unit of output they want to produce. Moreover, output takes one period so that entrepreneurs must pay one period's interest on their labor costs.[1] The entrepreneurs hence borrow from the banking system, which operates on a pure "overdraft" basis. That is, banks grant all needed loans at a given interest rate and in turn borrow needed reserves from the central bank (terminology from Hicks 1974; for modern accounts, see Moore 1988; Lavoie 1992; Wray 1998). Mathematically, these assumptions mean that costs are:

$$C=(1+R).a.W$$

Here and in subsequent equations, capital letters indicate variable names. Small letters are positive parameters or function names. Periods indicate multiplication. Small letters followed immediately by parentheses indicate functions. W is the hourly wage, a is the (fixed) labor/output ratio, R is the interest rate, and C is the cost of production of one unit of output.

The story of the equation is as follows. Firms hire workers at the beginning of the period. They start with no money, but must pay wages in advance. Therefore, they borrow the full amount of their wage bill from banks. The central bank sets the interest rate for reserve borrowings and banks, which, for convenience, charge no markup. Firms pay workers the full amount of their wages, which are a.W times the amount produced.

[1] Structuralists have emphasized the role of working capital in constructing models of the stagflationary effects of monetary contractions (Taylor 1983).

At the end of the period, firms sell their output. We will see that they charge a markup on their costs. For each unit of output, these costs include the wage bill, a.W and one period's interest on the loan, r.a.W. The firm's equilibrium receipts are a gross markup, m, times total unit costs (1+r).a.W, times the number of units sold. In a moment, we see how prices adjust to their equilibrium level of m times unit costs. After receiving its sales proceeds, the firm pays back its loan by making a deposit of (1+r).a.W. They retain a profit of (m-1).(1+r).a.w times units sold.

Using the approximation that the logarithm of (1+R) is roughly R for small R, the equation for the logarithm C is:

$$\mathrm{Ln}(C) \approx R + \ln(a) + \ln(W) \qquad (1)$$

where the function ln indicates the natural logarithm. It is assumed that prices adjust toward a fixed markup over costs, in a Kaleckian way.

$$\Pi = b.(\ln(m.C) - \ln(P)) \qquad (2)$$

where Π is inflation ((dP/dt)/P), and P is the price level. Substituting (1) into (2) and differentiating by time, one gets:

$$d\Pi/dt = b.((dR/dt) + (dW/dt)/W - \Pi) \qquad (3)$$

where the approximation is replaced by an equals sign for convenience. To flesh out the details of this equation, one assumes the following.

$$R^{*}=c(\Pi-\Pi^{*},Y-Y^{P})$$

where R* is the central bank's target interest rate, Y equals output or income, Π^* and Y^P are target levels, and c_Π, $c_Y > 0$, using subscripts to indicate partial derivatives. The function c and all other functions are assumed to be smooth. The central bank determines the optimal interest rate as a function c of how far Y and Π diverge from their target levels. The function c is a very general form of a J. Taylor (1993) rule, which is often used largely as an ad-hoc construct in macroeconomic models. Interest rates are adjusted by the central bank according to its preferences regarding inflation and output levels. Next, we posit a process by which the central bank adjusts the policy interest rate toward its target level, R*.

$$dR/dt = h.(R^* - R) \qquad (4)$$

The constant h is a speed of adjustment of the interest rate toward the target given by the Taylor rule. Judd and Rudebusch (1998) have used a somewhat similar adjustment equation in connection with a Taylor rule, arguing “Central banks often appear to adjust interest rates in a gradual fashion—taking small, distinct steps toward a desired setting.” The wage growth equation is:

$$(dW/dt)/W = d(Y) + \Pi \qquad (5)$$

$d_Y > 0$. Wages are driven by the power of labor, which is positively affected by a vigorous economy (high Y). The inflation term indicates that there is no “money illusion” in the wage bargaining process; it can be thought of as a “myopic perfect foresight” expectations term (Flaschel, Franke, and Semmler 1997). Since workers know the current derivative of the price level, any existing inflation does not affect the outcome of the bargaining process in real terms. Though different assumptions about expectations formation may have important implications, these are somewhat extraneous to the issues examined in this paper. We do not wish to explore further how incorrect expectations of the price level might affect the business cycle, a hypothesis that has been amply discussed elsewhere.

Plugging (4) and (5) into (3):

$$d\Pi/dt = b.(h.(c(\Pi-\Pi^*, Y-Y^P)- R) + d(Y)) \qquad (6)$$

The demand side of the model features an "acceleration channel"— the rate of change of the interest rate affects output (Minsky 1986 and Hannsgen 2005; see Minsky quotations within the latter). Interest rate changes can have an effect on the financial condition of banks and other firms, particularly when there is maturity mismatch between their assets and liabilities. Perhaps one of the most extreme cases is the savings-and-loan crisis of the late 1970s and early 1980s. As Federal Reserve Board Chairman Paul Volcker raised interest rates, savings and loans found themselves losing deposits to instruments with a greater return than deposits. Eventually, savings and loan institutions were able to raise their deposit rates in an effort to retain funds. But the assets of savings and loan associations were mainly fixed-rate, long-term mortgages. Thus, even as savings and loans paid more for deposits, their income remained largely unchanged. The resulting squeeze was one factor that ultimately led to the loss of all positive net worth of the industry. Of course, it is admitted that the rate of change of the interest rate is not the same as the difference between short and long rates, but the acceleration term in aggregate demand is a simple way to capture the more complicated real-world phenomenon.

Nonetheless, a stylized stochastic model can be developed in which the two concepts are identical.[2]

To reflect these considerations, it is assumed that part of real aggregate demand (or sales) is a linear function of output and part is a function of the (expected) real interest rate *and* the rate of change of the interest rate.

$$D = k + n.Y + e(dR/dt, R\text{-}\Pi)$$

$0 < n < 1$

In accordance with the argument of the previous paragraphs, $e_{dR/dt}<0$ and $e_R<0$. The Π term fits with the previous assumption of myopic perfect foresight. The mechanism equilibrating demand and supply is

$$dY/dt = g.(D - Y)$$

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[2] Suppose there are two interest rates:

R_t is the short period rate in period t. I can lend one dollar in period t and get back $1+R_t$ dollars in period t+1.

r_t is the long-term (two period) interest rate in period t. I can lend one dollar in period t and get back $1+r_t$ dollars in period t+2.

I hypothesize that the short-term interest rate follows a random walk:

$R_{t+1} = R_t + e_{t+1}$

Where e_{t+1} is a random, serially uncorrelated variable with a mean of zero.

Then:

$E_t(R_{t+1}) = R_t$ (1)
where E is the expectations operator. Now suppose I assume an "expectations" theory of the determination of the interest rate. The two-period (long-term) interest rate is the average of this period's short-term rate and the expectation of next period's short-term rate. Using (1),

$r_t = (E_t(R_{t+1}) + R_t)/2 = R_t$ (2)

Then, shifting (2) back by one period, one can see that the difference between the current cost of funds and the "old," long-term loans on the books is the same as the first difference of the short rate:

$R(t)\text{-}r(t\text{-}1) = R(t) - R(t\text{-}1)$
.

$$= g.(k + n.Y + e(dR/dt, R\text{-}\Pi) – Y)$$

$$= g.(k + n.Y + e(h.(c(\Pi\text{-}\Pi^*, Y\text{-}Y^P) – R), R\text{-}\Pi) - Y) \quad (7)$$

It is assumed that excess or deficient demand results in an undesired change in inventories, so that demand always equals sales, if not output (Y).

4. An Analysis of the Dynamics

The dynamics can be previewed briefly. The system at issue is given by differential equations (4), (6), and (7), each in the variables Π, Y, and R. First, the Gibson effect, as one might have foretold, has a tendency to destabilize the equilibrium point. Moreover, under certain, fairly weak, assumptions, it will be shown, the model fits locally into a well-known "genus" of cycles, which has recently been explored by L. Taylor (2004) and the Bielefeld School (for example, Flaschel, Franke, and Semmler 1997; Chiarella and Flaschel 2000). This can be shown by the Hopf bifurcation theorem. If three conditions hold, the only unknown is whether the local cycle is stable or unstable (subcritical or supercritical). Finally, very low or very high values of the sensitivity of policy to inflation guarantee instability.

The formalities are as follows:

PROPOSITION 1

ASSUMPTIONS:

(1) $e_R < h.e_{dR/dt}$

(2) $h.c_{\Pi}.e_{dR/dt} < e_R$

(3) the "nonzero speed" (non-inflection point) condition[3]

[3] This assumption involves a complicated equation involving all of the parameters *not* holding exactly. Details are available from the author. See below.

with all derivatives evaluated at the equilibrium point of the system, which is assumed to exist and be unique. Then the equilibrium point of (4), (6), (7) loses asymptotic stability as b increases above a certain threshold level, through the (local) birth of a stable limit cycle or the death of an unstable limit cycle. (Further bifurcations may occur, creating more cycles.)

PROOF (can be skipped):

The general necessary and sufficient conditions for stability of a 3x3 system of differential equations are (Gandolfo 1997, pp. 251-52):

STABILITY CONDITIONS:

(1) Trace (J) < 0, where J is the Jacobian (first derivative matrix) of the system:

$$J=\begin{bmatrix} b.h.c_{\Pi} & b.(h.c_Y + d_Y) & -b.h \\ g.(h.c_{\Pi}.e_{dR/dt} - e_R) & g.(n-1+h.c_Y.e_{dR/dt}) & g.(e_R - h.e_{dR/dt}) \\ h.c_{\Pi} & h.c_Y & -h \end{bmatrix}$$

(2) Det (J) < 0

(3) $$J^{*}=\begin{vmatrix} a_{11}+a_{22} & a_{23} & -a_{13} \\ a_{32} & a_{11}+a_{33} & a_{12} \\ -a_{31} & a_{21} & a_{22}+a_{33} \end{vmatrix} < 0$$

where a_{11} is the first element of the first row of J, a_{12} is the second element of the first row, etc. and the vertical lines indicate a determinant of the matrix within.

We shall see that the three conditions above are met under assumptions 1 and 2 for a sufficiently low value of b:

(1) the upper left term in Trace (J) is positive and the other two diagonal elements are negative. The upper left term can be made as small as desired by reducing b, so for sufficiently low b, this condition is met.

(2) By a series of factorizations and elementary row operations, it can be seen that the determinant of J is $b.g.h.d_Y.(c_\Pi-1).e_R$. It is clear that this determinant is negative for $c_\Pi>1$, which is guaranteed by assumptions 1 and 2. Note that this is true regardless of the value of b. Hence, condition 2 is met.

(3) The relevant matrix for condition 3 is:

$$J^*=\begin{vmatrix} b.h.c_\Pi + g.(n-1+e_{dR/dt}.h.c_Y) & g.(e_R - h.e_{dR/dt}) & b.h \\ h.c_Y & h.(b.c_\Pi - 1) & b.(h.c_Y + d_Y) \\ -h.c_\Pi & g.(h.c_\Pi.e_{dR/dt} - e_R) & g.(n-1+e_{dR/dt}.h.c_Y) - h \end{vmatrix}$$

The sign pattern of this matrix under assumptions 1 and 2 and for low b is:

$$\begin{vmatrix} - & - & + \\ + & - & + \\ - & - & - \end{vmatrix}$$

Note that the signs of the (1,1) and (2,2) elements of J* depend upon the smallness of b. The signs of the (1,2) and (3,2) elements rely on assumptions 1 and 2, respectively. I expand this determinant around the first row. The terms of the expansion corresponding to the (1,1) and (1,3) elements are unambiguously negative, given the sign pattern above. The (1,2) term of the expansion is:

$$- g.(e_R - h.e_{dR/dt}).([h.c_Y.(g.(n-1+e_{dR/dt}.h.c_Y) - h)]+[h.c_\Pi.b.(h.c_Y + d_Y)])$$

The first part (to the left of the first square bracket) is positive under Assumption 1. The first term in square brackets is negative by our assumptions on signs (and n<1).

The term in the second set of square brackets is positive, but it can be made arbitrarily small by making b sufficiently small. Thus, the expression as a whole is negative.

Hence, for sufficiently small b, all three stability conditions above are met. Now, note that if we increase b, the first condition (and, by implication, the third) is no longer met. Since the first condition is necessary for stability, we then know that the equilibrium becomes unstable for relatively high values of b. [The real (parts of the) roots go from negative to positive.] To prove a Hopf bifurcation, I must show that as b increases, two roots cross the imaginary axis with nonzero speed (Gandolfo 1997, pp. 475-79). It can be shown that the roots are continuous functions of b, allowing an application of the intermediate value theorem. Since the stability condition goes from being met to not being met and since the determinant of J (which is the product of the roots of J) stays nonzero as b is increased, we know that the loss of stability does not occur through a change from two real negative roots to two positive real roots (this would entail a zero determinant at the point of crossing). Hence, we know that the loss of stability occurs when two of the roots cross the imaginary axis from left to right, while the third root remains real and negative. [See Benhabib and Miyao 1981 for a mathematically similar proof with more details. In the appendix to that article, it is shown how to prove the "nonzero speed" part of the Hopf theorem, which is not really restrictive. (See Assumption 3)].

PROPOSITION 2

For $c_{\Pi} < 1$, the system described above is not stable.

PROOF:

Stability condition 2 in Proposition 1 is that det J < 0. The determinant is

$b.g.h.d_Y.(c_\Pi-1).e_R$

This expression is positive if $c_\Pi < 1$. Since Condition 2 is a necessary condition for stability, the proposition easily follows.

PROPOSITION 3

For c_Π sufficiently *large,* given other parameters, the system is not stable.

PROOF:

Stability condition 1 in Proposition 1 says that the trace of J must be negative. The (1,1) term of J is positive and can be made as large as necessary by increasing c_Π. The (2, 2) and (3, 3) elements of J do not involve this parameter. Since stability condition 1 is a necessary condition for stability, the result follows.

What is the economic meaning of the dynamics? Assumption 1 requires that the conventional interest rate effect in the "IS" curve be relatively strong compared to the term based on the rate of change of the interest rate. This should be a cautionary note about the desirability of this innovation, but Minsky and other people who have proposed this mechanism have often asserted that it generates instability, anyway. Assumption 2 indicates that c_Π, the sensitivity of the central bank's reaction function to inflation, be fairly strong. This is a well-known condition for stability of models in which a Taylor rule appears (Proposition 2). However, c_Π, *as it appears in the inflation equation*, is part of one of the positive terms in the trace of J (the 1,1 element) and so contributes to the *eventual* generation of instability once it gets *too* high (Proposition 3). So, highly responsive policy has a destabilizing effect insofar as interest rates appear in the price equation. (A sufficiently strong "Gibson effect", given constant values of all the other parameters, eventually creates instability.) Finally, the conditions for a cycle are not extremely restrictive.

SUMMARY AND CONCLUSIONS

The implication of the foregoing analysis is that one must be aware of possible perverse effects in implementing monetary policy. The model above incorporates a positive impact of interest rates on prices, with the novel addition of a central bank policy-setting function and a Minskyan AD curve that makes output a function of both the level and the rate of change of the interest rate.

An effect arising jointly from the inclusion of interest among the costs of production and the central bank's response to inflation leads to instability for certain parameter values. An important policy implication of the stability condition is that sufficiently high or low levels of the derivative with respect to inflation in the interest-rate-setting function always destabilize the model. A moderate value for the inflation parameter in the Taylor rule therefore seems desirable from a policy point of view. A complicated set of effects is involved, but the key causal chain is that an increase in inflation increases the central bank's tendency to raise rates, which only exacerbates the original inflationary problem.

Assuming the satisfaction of three conditions on the model parameters, some form of local cycle exists in inflation-output-interest rate space. The conditions arise, from the policy function, as mentioned, and from the relative strength of the effects of the interest rate and the rate of change of the interest rate on the demand side. The cycle may take the form of a “corridor of stability”, within which all paths lead to the equilibrium, or a stable limit cycle, to which all paths lead.

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