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Inequality of Life Chances and the Measurement of Social Immobility

by

Jacques Silber
Department of Economics, Bar-Ilan University
52900 Ramat-Gan, Israel
jsilber_2000@yahoo.com

Amedeo Spadaro
PSE (Paris-Jourdan Sciences Economiques,
Joint Research Unit 8545 CNRS-EHESS-ENPC-ENS), Paris;
FEDEA, Madrid; and
Universitat de les Illes Balears, Palma de Mallorca
spadaro@pse.ens.fr

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The Levy Economics Institute
P.O. Box 5000
Annandale-on-Hudson, NY 12504-5000
http://www.levy.org

ABSTRACT

This paper begins by proposing two cardinal measures of inequality in life chances as well as an ordinal representation of such inequality based on the use of so-called social immobility curves. Using as its database a matrix in which the lines correspond to the social category of parents (e.g., their occupation or educational level) and the columns to the income distribution of their children, it then highlights the importance of the marginal distributions when comparing social immobility within two populations, and shows how it is possible to neutralize differences in these margins. The idea is to adapt a method used in the field of occupational segregation measurement that allows one to make a distinction between differences in gross and net social immobility, assuming that the marginal distributions of the two populations are identical. Borrowing ideas from recent literature on the equality of opportunity, the paper then defines the concept of an inequality in circumstances curve and relates it to that of a social immobility curve.

Two empirical datasets are used to determine the usefulness of the concepts presented. The first dataset comes from a survey conducted in France in 1998 and allows one to measure the degree of social immobility and of inequality in circumstances on the basis of the occupation of fathers or mothers and the income class to which sons or daughters belong. The second dataset, drawn from a social survey conducted in Israel in 2003, is the basis for a study of social immobility and inequality in circumstances, emphasizing the transition from the educational level of the fathers to the income class of the children. Both illustrations confirm the usefulness of the analytical tools described in this paper.

Keywords: Equality of Opportunity, France, Inequality in Circumstances, Inequality in Life Chances, Israel, Social Mobility

JEL Classifications: D31, D63, I31, J62

1. INTRODUCTION

The topic of intergenerational mobility measurement has been increasingly popular among economists in recent years (for interesting surveys see, for example, Solon 1999; Corak, Gustafsson, and Österberg 2004). In a recent paper, Van de gaer, Schokkaert, and Martinez (2001) have made a distinction between three meanings of intergenerational mobility, stressing, respectively, the idea of movement, the inequality of opportunity, and the inequality of life chances.

When mobility is viewed as movement, the goal is usually to measure the degree to which the position of children is different from that of their parents. Those adopting such an approach tend to work with transition matrices whose typical element p_{ij} refers to the probability that a child has position j, given that his/her parents had position i. These matrices are square matrices and it should be clear that mobility in this sense will be higher the smaller the value of the probabilities p_{ii}.

The second approach to mobility attempts to measure the degree to which the income prospects of children are equalized, that is, they do not depend on the social origin of the parents. What is often done in such types of analyses when comparing, for example, two groups of parents (e.g., two educational levels or two occupations) is to check whether the distribution of the outcomes (incomes) of the children of one group of parents stochastically dominates the distribution of the second category of parents (see, for example, Peragine 2004; Lefranc, Pistolesi, and Trannoy 2006).

The third approach emphasizes the concept of inequality of life chances so that "the only thing that matters here is that children get equal lotteries. The prizes do not matter." (Van de gaer, Schokkaert, and Martinez 2001). Such an approach is particularly relevant when one analyzes the movement between socioeconomic categories which cannot always be ranked in order of importance.

In the present paper we focus most of our attention on the third approach and first propose two cardinal measures of social immobility (inequality in life chances) that do not require the matrix to be analyzed to be a square matrix. This allows us, for example, to study the transition from the original social category (educational level or occupation) of the parents to the income class to which the children belong. But these measures could also be applied to an analysis of the transition from one type of social category (e.g., educational level of the parents) to another type of social category (profession of the children), assuming there is no specific ordering of these categories. We also propose an ordinal representation of this approach, based on the use of what we have called “social immobility curves.”

In the second stage of the analysis we stress the importance of the marginal distributions when comparing social immobility in two populations. What we want to emphasize is that it makes sense to neutralize the differences between the two populations concerned in the distribution of the parents by social category (e.g., occupation or educational level) and the income distribution of the children. More specifically, we suggest borrowing a method that has been used in the field of occupational segregation measurement to make a distinction, when comparing two populations, between differences in gross and net social immobility, the latter concept assuming that the marginal distributions of the two populations compared are identical.

In a third stage, borrowing some ideas of the recent literature on the equality of opportunity (e.g., Roemer 1998; Kolm 2001; Ruiz-Castillo 2003; Villar 2005), we define the concept of inequality in the circumstances curve and relate it to the social immobility curve previously presented.

Two empirical illustrations then attempt to illustrate the usefulness of the concepts previously defined. The first set of data comes from a survey conducted in France in 1998 and allows us to measure the degree of social immobility and inequality in circumstances on the basis of information on the occupation of fathers or mothers and of the income class to which sons or daughters belong. Using a second set of data, based on a social survey conducted in Israel in 2003, we then measure social immobility and inequality in circumstances by studying the transition from the educational level of the fathers to the income class to which the children belong. Both illustrations seem to confirm the usefulness of the tools of analysis described in this paper. Concluding comments are given at the end of the paper.

2. MEASURING SOCIAL IMMOBILITY

2.A. Cardinal Measures of Social Mobility

2.A.1. Defining Two Indices of Social Immobility

Let us assume a data matrix (M) whose lines (i) correspond to the social origin of the individuals (e.g., occupation of the father or mother, or educational level of the father or mother) and whose columns (j) correspond to the income brackets to which these individuals belong. For example, M_{ij} would give the number of individuals whose income belongs to the income bracket j and whose father had educational level i.

Define now m_{ij} as $m_{ij} = M_{ij} / (\sum_{i=1 \text{ to } I} \sum_{j=1 \text{ to } J} M_{ij})$, $m_{i.}$ as $m_{i.} = (\sum_{j=1 \text{ to } J} m_{ij})$, and $m_{.j}$ as $m_{.j} = (\sum_{i=1 \text{ to } I} m_{ij})$.

Perfect social mobility may be assumed to exist when the probability that an individual belongs to a specific income bracket (k) is independent of his social origin (h) (e.g., educational level of his father). In other words, in such a case we could write that $m_{hk} = (m_{h.}\, m_{.k})$. As a consequence, any index measuring the degree of independence between the lines and the columns of such a matrix could be selected as a measure of social mobility.

A first measure one may think of is an entropy related index such as one of Theil's (1967) famous indices which amount somehow to comparing "prior probabilities" with "posterior probabilities." In our case the "prior probabilities" would be the products $m_{h.}\, m_{.k}$, while the "posterior probabilities" would be the proportions m_{hk}. Such a formulation of the Theil index would be

$$T_{sim} = \sum_{i=1 \text{ to } I} \sum_{j=1 \text{ to } J} \{(m_{i.}\, m_{.j}) \ln [(m_{i.}\, m_{.j})/m_{ij}]\} \qquad (1)$$

The subscript "sim" in T_{sim} in (1) indicates that such an index would, in fact, be a "social immobility index" because it would be equal to 0 when there is perfect independence between the social origins and the income brackets.

Another possibility is to use a Gini-related index, as suggested originally by Flückiger and Silber (1994). As stressed also by Silber (1989a), the Gini index may be also used to measure the degree of dissimilarity between a set of "prior probabilities" and a set of "posterior probabilities." In the case of inequality measurement, the "prior probabilities" are the population shares and the "posterior probabilities" the income shares, while when measuring occupational segregation by gender, the "prior probabilities" are, for example, the shares of male workers employed in the various occupations and the "posterior probabilities" the corresponding shares of female workers (or the reverse).

Such a Gini-related index of social (im)mobility may be expressed as

$$G_{sim} = [\ldots(m_{i.}\, m_{.j})\ldots]' \; G \; [\ldots m_{ij} \ldots] \qquad (2)$$

where $[\ldots(m_{i.}\, m_{.j})\ldots]'$ is a row vector giving the "prior probabilities" corresponding to the various (I×J) cells (i,j), while $[\ldots\ldots m_{ij} \ldots\ldots]$ is a column vector giving the "posterior probabilities" (the actual probabilities) for these cells. Note that, as indicated in Silber (1989a), the elements of these row and column vectors have both to be ranked by decreasing ratios $(m_{ij})/(m_{i.}\, m_{.j})$. The operator G in (2), called G-matrix (see Silber 1989b), is a (I×J) by (I×J) square matrix whose typical element g_{pq} is equal to 0 if p=q, to -1 if p>q, and to +1 if p<q.

Note that the index G_{sim} is also a social immobility index because it will be equal to zero when all "prior probabilities" ($m_{i.}$ $m_{.j}$) are equal to the "posterior probabilities" m_{ij} and in such a case we would have perfect mobility. However, since the Gini index is bounded by 0 and 1, we can define a social mobility index G_{sm} as

$$G_{sm} = 1 - G_{sim} \tag{3}$$

2.A.2. Properties of the Social Mobility Indices

2.A.2.a. The Theil social immobility index

2.A.2.a.1. Value of the index when there is perfect immobility. We know that when there is perfect mobility $m_{ij} = (m_{i.}\, m_{.j})$ $\forall$i and $\forall$j, so that T_{sim} will be equal to zero.

2.A.2.a.2. Impact of two "margin-preserving" transfers. In Appendix A we prove that if $(m_{fh}/(m_{f.} m_{.h})) > (m_{fk}/(m_{f.} m_{.k}))$ and $(m_{lk}/(m_{l.} m_{.k})) > (m_{lh}/(m_{l.} m_{.h}))$ and if two "transfers" of size δ take place, one from m_{fh} to m_{fk} and one from m_{lk} to m_{lh}, so that the combination of these two transfers implies that there was no change in the marginal shares, the value of the Theil index of immobility will decrease.

2.A.2.b. The Gini social immobility index

2.A.2.b.1. Value of the index when there is perfect immobility We know that when there is perfect mobility $m_{ij} = (m_{i.}\, m_{.j})$ $\forall$i and $\forall$j. Given the definition of the G-matrix given previously, it is easy to see that in such a case the Gini immobility index G_{sim} will be equal to zero.

2.A.2.b.2. Impact of two "margin-preserving" transfers Assume we order the products ($m_{i.}$ $m_{.j}$) of the elements ($m_{i.}$) and ($m_{.j}$) by increasing values of the ratios $(m_{ij}/(m_{i.}\, m_{.j}))$. Let us similarly order the shares (m_{ij}) by increasing values of the ratios $(m_{ij}/(m_{i.}\, m_{.j}))$. We now plot the cumulative values of the elements ($m_{i.}$ $m_{.j}$) on the horizontal axis and the cumulative values of the shares (m_{ij}) on the vertical axis. The curve obtained will be called a "social immobility curve." It is, in fact, what is known in the literature as a relative concentration curve and clearly its slope is nondecreasing.

In the specific case when $m_{ij} = (m_{i.}\, m_{.j})$ $\forall$i and $\forall$j, this curve will become the diagonal line going from (0,0) to (1,1).

Assume now that $(m_{fh}/(m_{f.}m_{.h}))>(m_{fk}/(m_{f.}m_{.k}))$ and that $(m_{lk}/(m_{l.}m_{.k}))>(m_{lh}/(m_{l.}m_{.h}))$. Assume also that two "transfers" of size δ take place, one from m_{fh} to m_{fk} and one from m_{lk} to m_{lh}, so that the combination of these two transfers implies that there was no change in the marginal shares. It should be clear that in such a case the curve we have just defined will get closer to the diagonal line so that the area between this curve and the diagonal will become smaller. However, since the definition of the Gini social immobility index G_{sim} in (2) was a simple extension of the traditional definition of the Gini income inequality index (see Silber 1989b) to the case of social immobility measurement and since, in the former case, we know that the Gini index is equal to twice the area lying between the diagonal and the Lorenz curve, we can conclude that when the area lying between a "social immobility curve" and the diagonal decreases, the value of the social immobility index G_{sim} will decrease. Therefore, the sum of two "progressive margin preserving transfers" leads to a decrease in the value of the Gini immobility index G_{sim}.

2.B. An Ordinal Approach to Measuring Social Immobility: Drawing "Social Immobility Curves"

We can first draw "Social Immobility Curves" as they have been defined previously in Section 2.A.2.b.2. Using results that have appeared in the income inequality literature we can conclude, when comparing social immobility in two populations, if in subpopulation A the "social immobility curve" lies nowhere below but at times lies above that corresponding to subpopulation B, social immobility in subpopulation A is smaller than that in subpopulation B.

3. COMPARING TWO SOCIAL MOBILITY MATRICES

Assume now that we want to compare two social mobility matrices $\{m_{ij}\}$ and $\{v_{ij}\}$. Such matrices may refer to two different periods or to two population subgroups at a given time, such as ethnic groups, regions, etc. On the basis of each of these two matrices we could compute social immobility indices such as those defined in expressions (1) and (2), and conclude in which case social immobility is higher. This may, however, be too hasty a way to draw firm conclusions, as will now be shown.

Assume the social mobility matrices $\{m_{ij}\}$ and $\{v_{ij}\}$ correspond to two different time periods. The point is that social mobility may vary over time for various reasons. First, there may have been a change in the distribution of parents by social origin. Second, there may have been a change in the income distribution of the children. These two possibilities correspond evidently to a variation in one of the margins of the social mobility matrix. There may, however, be a third reason for a variation in the degree of social mobility. Even if there was no change over time in the margins of the social mobility matrix, there may have been a change in the degree of independence between the rows (social origin of the parents) and columns (shares of various income brackets) of this matrix. This distinction between the impact of a change in the margins of a matrix and a change in the "internal structure" of this matrix [this is the terminology used by Karmel and MacLachlan (1988) when they refer to the degree of independence between the lines and columns of a matrix] was pointed out by Karmel and MacLachlan (1988) in the framework of an analysis of changes over time in occupational segregation by gender.

The purpose of this section is to show that it is possible to apply the methodology proposed by Karmel and Maclachlan (1988) when analyzing changes over time in the degree social mobility (or when comparing the degree of social mobility of two population subgroups). Such a methodology uses an algorithm originally proposed by Deming and Stephan (1940). In addition, it will be shown [as was already stressed by Deutsch, Flückiger, and Silber (2006) in the framework of occupational segregation analysis] that it is possible to generalize this methodology by applying also the concept of Shapley decomposition (see Chantreuil and Trannoy 1999; Shorrocks 1999; Sastre and Trannoy 2002). A simple presentation of this approach is given in Appendix B. As far as its application to social mobility analysis is concerned, we will be able to derive (see, Appendix C for the exact demonstration) the specific contributions to the overall variation in the extent of social immobility of changes in the income distribution of the individuals (the children) and in the distribution of the parents by social category (these two changes correspond to variations in the margins of the social mobility matrix), as well as in the "pure" variation in the extent of social immobility (changes in the degree of independence between the lines and columns of the social mobility matrix itself).

3.A. Neutralizing the Specific Impacts of Changes in the Margins of the Social Mobility Matrix

The methodology to be used to isolate the specific effects of changes in the margins is borrowed from the literature on the measurement of occupational segregation (see Karmel and McLachlan 1988) and is extended here to the measurement of variations in the extent of social mobility.

As stated previously, assume that the population under scrutiny is divided into I categories of social origin and J income brackets. We can now define a matrix $\{q_{ij}\}$ in such a way that its typical element (q_{ij}) is equal to the product ($m_{i.}\, m_{.j}$) of the margins i and j of the matrix $\{m_{ij}\}$.

Clearly if m_{ij} is equal to q_{ij} for all i and j, there is independence between the social origin (the lines of the matrix $\{m_{ij}\}$) and the income of the individuals (the columns of the matrix $\{m_{ij}\}$). If m_{ij} is not equal to q_{ij} for at least some i and j, the rows and the columns are at least partially dependent and such a link between the income of an individual and his/her social origin should help us measure the extent of social immobility.

Let us now assume that we wish to decompose the variation over time in the extent of social mobility. In order to do so we will adopt a technique originally proposed by Deming and Stephan (1940) and used by Karmel and McLachlan (1988). The idea, when comparing two matrices of proportions $\{m_{ij}\}$ and $\{v_{ij}\}$, is to build a third matrix $\{s_{ij}\}$ which would have, for example, the internal structure of the matrix $\{m_{ij}\}$ but the margins of the matrix $\{v_{ij}\}$. To derive $\{s_{ij}\}$, one multiplies first all the cells (i,j) of $\{m_{ij}\}$ by the ratios $(v_{i.}/m_{i.})$ where $v_{i.}$ and $m_{i.}$ are, respectively, the horizontal margins of the matrices $\{v_{ij}\}$ and $\{m_{ij}\}$. Call $\{r_{ij}\}$ the matrix you derived after such a multiplication. Multiply now all the cells (i,j) of this matrix $\{r_{ij}\}$by the ratios $(v_{.j}/r_{.j})$ where $v_{.j}$ and $r_{.j}$ are now the vertical margins of the matrices $\{v_{ij}\}$and $\{r_{ij}\}$. Call $\{u_{ij}\}$ the new matrix you just derived. If you renew this procedure several times, the matrix you derive will quickly converge, as shown by Deming and Stephan (1940), to a matrix $\{s_{ij}\}$ that has the margins of the matrix $\{v_{ij}\}$ but, in a way, the internal structure of the matrix $\{m_{ij}\}$. In other words, the degree of social immobility corresponding to the matrix $\{s_{ij}\}$ is identical to that corresponding to the original matrix $\{m_{ij}\}$, but the matrix $\{s_{ij}\}$ has the same “income distribution of the individuals” and the same “structure of the social origin of these individuals” as that of the matrix $\{v_{ij}\}$. One could naturally have proceeded in the reverse order by starting with the matrix $\{v_{ij}\}$ and ending up with a matrix $\{w_{ij}\}$ that would have the margins of the matrix $\{m_{ij}\}$, but the internal structure of the matrix $\{v_{ij}\}$.

3.B. Decomposing Variations Over Time in the Extent of Social Mobility

In the previous section we have shown how the “move” from the matrix $\{m_{ij}\}$ to the matrix $\{v_{ij}\}$ included really two stages: one in which the margins were changed and one in which the internal structure of the matrix was modified. Let $\Delta SIM = SIM(v) – SIM(m)$ refer to the overall variation in the extent of social immobility. In Appendix C we first show, using the concept of Shapley decomposition (see Chantreuil and Trannoy 1999; Shorrocks 1999; Sastre and Trannoy 2002), that ΔSIM may be expressed as the sum of a contribution $C_{\Delta ma}$ of differences in the margins and a contribution $C_{\Delta is}$ of differences in the internal structure of the two social mobility matrices compared.

Then, applying the idea of a Nested Shapley decomposition (see Sastre and Trannoy 2002), we show that it is possible to further decompose the contribution $C_{\Delta ma}$ of the margins. More precisely, we show that this latter contribution $C_{\Delta ma}$ may itself be broken down into the sum of the contributions C_h and C_t of the horizontal and vertical margins.

4. MEASURES OF SOCIAL IMMOBILITY VERSUS MEASURES OF INEQUALITY IN CIRCUMSTANCES

Given that the case we are studying is that where the lines of the matrix to be analyzed correspond to "social origin" categories (e.g., occupation or educational level of the parents) and the columns to income classes (to which the "sons" or "daughters" belong), one may want to adopt the terminology used in the literature related to the measurement of equality of opportunity and call the lines "types" or "circumstances." Under certain conditions one may want to call the columns "levels of effort," although such an extension implies quite strong assumptions concerning the link between income and effort.

In any case, we will limit ourselves to attempting to derive a measure of inequality in circumstances. Adopting Kolm's (2001) ideas, we may define the inequality in circumstances as the weighted average of the inequalities within each "income class" ("effort level"), the weights being the population shares of the various income classes. We cannot, however, measure inequality the way Kolm (2001) suggested by comparing the average level of income for a given level of effort with what he calls the "equal equivalent" level of income for this same level of effort. We can, however, measure inequality within a given income class ("effort level") by comparing the distribution of the "actual shares" $(m_{ij}/m_{.j})$ for each income class (j) with what could be considered as the "expected shares" $(m_i/1)= m_{i.}$.

Using one of Theil's inequality measures this leads to the following measure of inequality within income class j:

$$T_j = \sum_{i=1 \text{ to } I} \{(m_{i.}) \ln [(m_{i.})/(m_{ij}/m_{.j})]\} \qquad (4)$$

The Theil measure of overall inequality in circumstances T_{circ} would then be defined as

$$T_{circ} = \sum_{j=1 \text{ to } J} (m_{.j})\, T_j = \sum_{j=1 \text{ to } J} (m_{.j}) \sum_{i=1 \text{ to } I} \{(m_{i.}) \ln [(m_{i.})/(m_{ij}/m_{.j})]\}$$

$$\leftrightarrow T_{circ} = \sum_{j=1 \text{ to } J} \sum_{i=1 \text{ to } I} \{(m_{.j}) (m_{i.}) \ln [(m_{i.})/(m_{ij}/m_{.j})]\}$$

$$\leftrightarrow T_{circ} = \sum_{j=1 \text{ to } J} \sum_{i=1 \text{ to } I} \{(m_{i.}) (m_{.j}) \ln [(m_{i.})(m_{.j})/(m_{ij})]\} \qquad (5)$$

which is, in fact, identical to the measure T_{im} of social immobility suggested in (1).

Let us now measure inequality in circumstances on the basis of the Gini index. Here again we will measure inequality within a given income class ("effort level") by comparing the distribution of the "actual shares" $(m_{ij}/m_{.j})$ for each income class (j) with what could be considered as the "expected shares" $(m_i/1)= m_{i.}$. Using the Gini-matrix which was defined in (2) we derive the following measure of inequality within income class ("effort level") j:

$$G_j = [\ldots(m_{i.})\ldots]' \; G \; [\ldots(m_{ij}/m_{.j})\ldots] \qquad (6)$$

$$\leftrightarrow G_j = (1/m_{.j}) \; [\ldots(m_{i.})\ldots]' \; G \; [\ldots(m_{ij})\ldots] \qquad (7)$$

where the two vectors (of length I) on both sides of the G-matrix in (6) and (7) are ranked by decreasing values of the ratios $(m_{ij}/m_{.j})/(m_{i.})$, that is, by decreasing values of the ratios $(m_{ij})/(m_{i.})$. To derive an overall Gini index of inequality of circumstances (G_{circ}) we will have to weight the indices given in (7) by the weights of the income classes j. We should however remember that in defining such an overall within groups Gini inequality index the sum of the weights will not be equal to 1 because each weight will in fact be equal to $(m_{.j})^2$ in the same way as in the traditional within groups Gini index the weights are equal to the product of the population and income shares. We therefore end up with

$$G_{circ} = \sum_{j=1 \text{ to } J} (m_{.j})^2 \, (1/m_{.j}) \; [\ldots(m_{i.})\ldots]' \; G \; [\ldots(m_{ij})\ldots]$$

$$\leftrightarrow \; G_{circ} = \sum_{j=1 \text{ to } J} (m_{.j}) \; [\ldots(m_{i.})\ldots]' \; G \; [\ldots(m_{ij})\ldots]$$

$$\leftrightarrow \; G_{circ} = \sum_{j=1 \text{ to } J} [\ldots,(m_{i.})(m_{.j}), \ldots]' \; G \; [\ldots(m_{ij})\ldots] \qquad (8)$$

Note that the formulation for G_{circ} in (8) is not identical to that of G_{sim} in (2). To see the difference between these two formulations, the following graphical interpretation may be given. As was done when drawing a social immobility curve, put respectively on the horizontal and vertical axes the expected shares $(m_{i.})$ and the actual shares (m_{ij}), starting with income class 1 and ranking both sets of shares by increasing ratios $(m_{ij})/(m_{i.})$. Then do the same for income class 2 and continue with the other classes until you end up with income class I. What we have then obtained is a curve which could be called an "inequality in circumstances" curve which comprises I sections, one for each income class. Clearly the slope of this curve is not always nondecreasing. It is nondecreasing within each income class but the curve reaches the diagonal each time we end with an income class.

We should however note that the shares used to draw such an "inequality in circumstances curve" are the same as that used in constructing a social immobility curve [compare both sides of the G-matrix in (2) and (8)]. In drawing the curve measuring inequality in circumstances we have simply "reshuffled" the sets of shares used in drawing a social immobility curve. Rather than ranking both sets of shares [on one hand the cumulative shares $((m_{i.})(m_{.j}))$, the cumulative shares (m_{ij}) on the other hand] by increasing values of the ratios $(m_{ij})/((m_{i.})(m_{.j}))$ working with all the I by J shares, we have first collected the shares corresponding to the first (poorest) income class and ranked them by increasing ratios $(m_{i1})/((m_{i.})(m_{.1}))$, and then did the same successively for all income classes.

An illustration of the difference between an "inequality in circumstances" curve and a social immobility curve is given in Figure 1 which will be analyzed in the empirical section. Note that whereas the index G_{circ} is equal to twice the area lying between the "inequality in circumstances" curve and the diagonal, the index G_{sim} is equal to twice the area lying between the social immobility curve and the diagonal. The area lying between the inequality in circumstances curve and the social immobility curve may then be considered as a measure of the degree of overlap between the various income classes in terms of the gaps between the "expected" and "actual" shares.

5. THE EMPIRICAL ANALYSIS

5.A. The Data Sources

We have analyzed two sets of data. The first set is a survey of 2000 individuals conducted in France by Thomas Piketty in the year 1998 with financial support from the McArthur Foundation (see Piketty 1999). The data set contains 65 variables and includes income, many sociodemographic characteristics, and answers to questions on social, political, ethical, and cultural issues.

Two types of variables were drawn from this database. To measure the social origin of the parents we used information on the profession of either the father or the mother. Eight professions were distinguished:

1) farmer (i.e., head of agricultural enterprise)
2) businessman, store owner, or "artisan"
3) manager or independent professional
4) technician or middle-rank manager
5) employee
6) blue collar worker, including salaried persons working in agriculture
7) not working outside the household
8) retired

The social status of the children's generation was measured via their monthly income classified in eight categories:

1) less than 4,000 FF
2) from 4,000FF to 5,999FF
3) from 6,000FF to 7,999FF
4) from 8,000FF to 9,999FF
5) from 10,000FF to 11,999FF
6) from 12,000FF to 14,999FF
7) from 15,000FF to 19,999FF
8) 20,000FF or more

On the basis of these two variables we built a social mobility matrix $\{m_{ij}\}$ whose lines (i) refer to the profession of either the father or the mother, depending on the case, and whose columns (j) correspond to the income group to which the individual (the child, either the son or the daughter, depending on the case examined) belongs. In other words, m_{ij} represents the share in the total population of those who belong to income group j and whose parents had profession i.

The second data set we worked with was the Social Survey that was conducted in Israel in 2003. The social origin of the individual was measured via the highest educational certificate or degree the father of the individual had received. Seven educational categories were distinguished:

1) Elementary school completion
2) Secondary school completion, but not a baccalaureate
3) Baccalaureate certificate
4) Post-secondary, nonacademic certificate
5) BA, academic certificate, or similar certificate
6) MA, MD, or similar certificate
7) PhD or similar diploma

The status of the children was measured via the total gross income of all members of the household to which the individual belonged, whatever the source of the income (work, pensions, support payments, rents, etc.). Ten income classes were distinguished:

1) NIS 2,000 or less[1]
2) NIS 2,001–3,000
3) NIS 3,001–4,000
4) NIS 4,001–5,000
5) NIS 5,001–7,000
6) NIS 7,001–9,000
7) NIS 9,001–12,000
8) NIS 12,001–15,000
9) NIS 15,001–20,000
10) More than NIS 20,000

On the basis of these two variables we built a social mobility matrix $\{m_{ij}\}$ whose lines (i) refer to the educational level of the father and whose columns (j) correspond to the income group to which the individual (the child) belongs. In other words m_{ij} represents the share in the total population of those who belong to income group j and whose fathers had educational level i.

We then analyzed differences in the degree of social mobility between three groups: the individuals whose father was born in Asia or Africa, those whose father was born in Europe or America, and those whose father was born in Israel.

5.B. The Results of the Empirical Investigation

5.B.1. Measuring Social Immobility

5.B.1.a. The French data The results of the analysis are reported in Table 1 which examines six different comparisons. In each comparison we give the value of the Gini social immobility index and decompose the difference between the values taken by this index in two different cases into the three components mentioned previously. For each index and component we also give confidence intervals based on a bootstrap analysis (see Appendix D for more details on this procedure). It is easy to observe that in all six tables (1-A to 1-F) the two Gini indices of social immobility which are compared are always significantly different one from the other. Moreover each of the components in all the six tables is always significantly different from zero.

The first striking result is that the degree of social immobility is quite higher when comparing fathers and sons (Gini social immobility index equal to 0.202) or fathers to daughters (Gini index equal to 0.193) than when comparing mothers to sons (Gini index equal to 0.143) or even mothers to daughters (Gini index equal to 0.166). We also observe when comparing, for example, the degree of social immobility from fathers to sons with that from mothers to sons that the difference (lower degree of social immobility in the latter case) is even much higher once we control for the margins. In other words, the difference in the degree of "net social immobility" is,

[1] NIS stands for New Israeli Shekels.

in this case, much higher (0.140 rather than 0.049 for the difference in “gross social immobility”). Note also that this impact of the margins is usually mainly one which is related to differences in the occupational structure of the parents.

In Figure 1 we have drawn the “Gross Social Immobility Curve” for the case where the transition analyzed is that from father’s occupation to daughter’s income. The second curve in Figure 1 will be analyzed in Section 5.B.2.a.

5.B.1.b. The Israeli data The results of the analysis are presented in Table 2, again with bootstrap confidence intervals. Note that here also in the three comparisons which are made the Gini indices of social immobility that are compared are significantly different from one another and each of the components is always significantly different from zero.

The most striking result here is certainly the fact that social immobility (mobility) is much higher (lower) among those who were born in Asia or Africa (Gini index of 0.233) than among those born in Europe or America (Gini index of 0.124), or even among those born in Israel (Gini index of 0.147).

The results are even more striking when we compare “gross” with “net” immobility using the algorithm described in Section 2.A.3. Thus, when looking at the results given in Table 2A we observe that whereas the “gross difference” between the Gini indices of social immobility of those born in Asia or Africa and those born in Europe or America is equal to 0.110, the “net difference” (net of changes in the margins) is equal to 0.310. Note also that this impact of the margins is essentially an impact of differences in the education levels of (the fathers of) the two groups compared.

Quite similar conclusions may be drawn when comparing individuals whose father was born in Asia or Africa and individuals whose father was born in Israel (Table 2B).

In Figure 2 we compare “Gross” and “Net Social Immobility Curves.” The “Gross” curves are drawn for two groups, corresponding respectively to the individuals whose father was born in Europe or America (EA) and Asia or Africa (AA). The “Net Social Immobility Curve” was drawn on the basis of a matrix which has the margins of the matrix of those born in Europe or America, but the “internal structure” of the matrix of those born in Asia or Africa. This evidence shows clearly how much bigger the gap in social immobility is when comparing the EA and AA groups once we control for the margins.

5.B.2. Inequality in Circumstances

5.B.2.a. The French data In Table 3 we give two examples of the use of the Theil or Gini indices of inequality in circumstances. The first example refers to French data where the data analyzed are those relative to the transition from the occupation of the fathers to the income class to which the daughters belong (see Table 3A and 3B). First we may note (see Table 3A) that, as expected, the sum of the contribution of the various income classes to the overall Theil index of inequality in circumstances is indeed equal to this latter index. Second we may note that there is some discrepancy between the results in Tables 3A and 3B. Whereas on the basis of the Theil index the two income classes where inequality in circumstances is highest are those corresponding, respectively, to the income ranges 10,000FF to 11,999FF and 15,000FF to 19,999FF, the two income classes where inequality in circumstances is highest, according to the Gini index of inequality in circumstances, are those corresponding, respectively, to the income ranges 4,000FF to 5,999FF and 20,000FF or more. As far as the highest relative contributions of the income classes to the overall Theil or Gini indices of inequality in circumstances are concerned (remember that the weights of the income classes are not the same for the Theil and the Gini index) the results are quite similar. In both cases the highest relative contribution is that of the richest income class. The second highest is that of the class with an income range of 4,000FF to 5,999FF for the Theil index and that of the classes 4,000FF to 5,999FF, as well as 12,000FF to 14,999FF (the results are almost identical for these two income classes) for the Gini index.

As mentioned previously we have drawn in Figure 1 the "Gross Social Immobility Curve" for the case where the transition analyzed is that from father's occupation to daughter's income. On this same graph we have plotted what was previously called a "Curve of Inequality in Circumstances." The large area lying between both curves indicates clearly that the gap between the "expected" shares ($m_{i.}\ m_{.j}$) and the "actual" shares m_{ij} is not a function of income, hence the great degree of overlapping between the income classes when the ranking is based on the ratio of actual over expected shares.

5.B.2.b. The Israeli data The second illustration is based on Israeli data concerning individuals whose father was born in Asia or Africa (see Table 3C). Unfortunately we could not compute Theil indices because some of the cells in the data matrix were empty. For the Gini index (see, Table 3-C) the two income classes with the highest values of this index are those corresponding to the income ranges NIS 3,001 to 4,000 and NIS 4,001 to 5,000. We may also observe that according to the Gini index (see Table 3C), the two income classes that contribute most to the overall value of the Gini index of inequality in circumstances are those corresponding to the income ranges from NIS 7,001 to 9,000 and 9,001 to 12,000.

6. CONCLUDING COMMENTS

This paper first suggested new tools of analysis to study intergenerational social immobility. More precisely, two indices measuring social immobility were proposed, derived from the Theil and Gini indices of inequality, and the concept of "social immobility curve" was introduced. The main contribution of this paper is, however, its stress of the need to make a distinction between concepts of "gross' and "net" social immobility and, hence, to emphasize the fact that when comparing social immobility in two groups it is probably better to base the comparison on a case where the margins of the matrices corresponding to the two groups are the same. In fact, it appears that measures of differences between two groups in "gross" versus "net" social immobility may sometimes lead to opposite conclusions. Finally, this paper also suggested two measures of inequality in circumstances, derived also from the Theil and Gini indices. Whereas the Theil index of inequality in circumstances turned out to be identical to the Theil index of social immobility, we showed that the Gini index of inequality in circumstances did not measure the same thing as the Gini index of social immobility and these differences were indeed confirmed by the empirical illustration.

TABLE 1. SOCIAL MOBILITY—RESULTS BASED ON THE FRENCH DATA BASE

A. Comparing Social Mobility from Fathers to Daughters with Social Mobility from Fathers to Sons, Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from fathers to daughters (G_1)	0.20237	0,0047	0,1947	0,2101
Social immobility from fathers to sons (G_2)	0.19261	0,0045	0,1852	0,2000
Difference (G_2 - G_1)	- 0.00976	0,0002	-0,0101	-0,0094
"Net" difference in social immobility	- 0.00083	0,0000	-0,0009	-0,0008
Difference due to difference in the margins of the social mobility matrix	- 0.00892	0,0002	-0,0093	-0,0086
Difference due to differences in the "professional composition" of the fathers' generation	- 0.00780	0,0002	-0,0081	-0,0075
Difference due to differences in the income distribution of the children (daughters versus sons)	- 0.00112	0,0000	-0,0012	-0,0011

B. Comparing Social Mobility from Fathers to Daughters with Social Mobility from Mothers to Sons, Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from fathers to daughters (G_1)	0.20237	0,0047	0,1947	0,2101
Social immobility from mothers to sons (G_2)	0.14332	0,0033	0,1380	0,1487
Difference (G_2 - G_1)	- 0.05905	0,0014	-0,0613	-0,0568
"Net" difference in social immobility	0.03358	0,0008	0,0323	0,0349
Difference due to difference in the margins of the social mobility matrix	- 0.09263	0,0021	-0,0961	-0,0891
Difference due to differences in the "professional composition" of the parents (fathers versus mothers)	- 0.08731	0,0020	-0,0906	-0,0840
Difference due to differences in the income distribution of the children (daughters versus sons)	- 0.00532	0,0001	-0,0055	-0,0051

C. Comparing Social Mobility from Fathers to Sons with Social Mobility from Mothers to Daughters, Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from fathers to sons (G_1)	0.19261	0.0045	0.1853	0.1999
Social immobility from mothers to daughters (G_2)	0.16609	0.0038	0.1598	0.1724
Difference ($G_2 - G_1$)	- 0.02652	0.0006	-0.0275	-0.0255
"Net" difference in social immobility	- 0.09566	0.0022	-0.0993	-0.0920
Difference due to difference in the margins of the social mobility matrix	0.06913	0.0016	0.0665	0.0718
Difference due to differences in the "professional composition" of the parents (fathers versus mothers)	0.07993	0.0018	0.0769	0.0830
Difference due to differences in the income distribution of the children (sons versus daughters)	- 0.01080	0.0002	-0.0112	-0.0104

D. Comparing Social Mobility from Fathers to Daughters with Social Mobility from Mothers to Daughters Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from fathers to daughters (G_1)	0.20237	0.0046	0.1947	0.2100
Social immobility from mothers to daughters (G_2)	0.16609	0.0039	0.1598	0.1724
Difference ($G_2 - G_1$)	- 0.03628	0.0008	-0.0377	-0.0349
"Net" difference in social immobility	0.04479	0.0010	0.0431	0.0465
Difference due to difference in the margins of the social mobility matrix	- 0.08107	0.0019	-0.0842	-0.0780
Difference due to differences in the "professional composition" of the parents (fathers versus mothers)	- 0.08163	0.0019	-0.0847	-0.0785
Difference due to differences in the income distribution of the daughters	0.00056	0.0000	0.0005	0.0006

E. Comparing Social Mobility from Fathers to Sons with Social Mobility from Mothers to Sons, Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from fathers to sons (G_1)	0.19261	0.0045	0.1852	0.2001
Social immobility from mothers to sons (G_2)	0.14332	0.0033	0.1379	0.1488
Difference (G_2 - G_1)	- 0.04929	0.0012	-0.0512	-0.0474
"Net" difference in social immobility	- 0.13980	0.0032	-0.1451	-0.1345
Difference due to difference in the margins of the social mobility matrix	0.09051	0.0021	0.0871	0.0940
Difference due to differences in the "professional composition" of the parents (fathers versus mothers)	0.09097	0.0021	0.0875	0.0944
Difference due to differences in the income distribution of the sons	- 0.00046	0.0000	-0.0005	-0.0004

F. Comparing Social Mobility from Mothers to Daughters with Social Mobility from Mothers to Sons, Using the Gini Social Immobility Index

Components of the Difference	Value of the Gini Social Immobility Index	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)
Social immobility from mothers to daughters (G_1)	0.16609	0.0039	0.1597	0.1725
Social immobility from mothers to sons (G_2)	0.14332	0.0033	0.1379	0.1487
Difference (G_2 - G_1)	- 0.02277	0.0005	-0.0236	-0.0219
"Net" difference in social immobility	- 0.01413	0.0003	-0.0147	-0.0136
Difference due to difference in the margins of the social mobility matrix	- 0.00864	0.0002	-0.0090	-0.0083
Difference due to differences in the "professional composition" of the mothers	- 0.00535	0.0001	-0.0056	-0.0051
Difference due to differences in the income distribution of the children (daughters versus sons)	- 0.00329	0.0001	-0.0034	-0.0032

TABLE 2. SOCIAL MOBILITY—RESULTS BASED ON THE ISRAELI DATA BASE

A. Comparing the Degree of Social Immobility among Those Whose Father Was Born in Europe or America and Those Whose Father Was Born in Asia or Africa, Using the Gini Social Immobility Index

Components of the Difference	**Gini Social Immobility Index**	**Bootstrap Standard Deviation**	**Conf. Int. Lower Bound (95%)**	**Conf. Int. Upper Bound (95%)**
Social immobility among those whose father born in Asia or Africa (G_1)	0.23276	0.0054	0.2239	0.2416
Social immobility among those whose father was born in Europe or America (G_2)	0.12357	0.0028	0.1189	0.1282
Difference (G_2 - G_1)	- 0.10919	0.0025	-0.1133	-0.1050
"Net" difference in social immobility	- 0.31028	0.0072	-0.3220	-0.2985
Difference due to difference in the margins of the social mobility matrix	0.20109	0.0047	0.1934	0.2088
Difference due to differences in the in the "educational composition" of the parents' generation	0.19625	0.0045	0.1889	0.2036
Difference due to differences in the income distribution of the children's generation	0.00484	0.0001	0.0047	0.0050

B. Comparing the Degree of Social Immobility among Those Whose Father Was Born in Asia or Africa and Those Whose Father Was Born in Israel, Using the Gini Social Immobility Index

Components of the Difference	**Gini Social Immobility Index**	**Bootstrap Standard Deviation**	**Conf. Int. Lower Bound (95%)**	**Conf. Int. Upper Bound (95%)**
Social immobility among those whose father born in Asia or Africa (G_1)	0.23276	0.0054	0.2239	0.2416
Social immobility among those whose father was born in Israel (G_2)	0.14696	0.0033	0.1415	0.1525
Difference (G_2 - G_1)	- 0.08580	0.0020	-0.0891	-0.0825
"Net" difference in social immobility	- 0.16360	0.0038	-0.1699	-0.1573
Difference due to difference in the margins of the social mobility matrix	0.07780	0.0018	0.0748	0.0808
Difference due to differences in the in the "educational composition" of the parents' generation	0.07953	0.0018	0.0765	0.0825
Difference due to differences in the income distribution of the children's generation	- 0.00173	0.0000	-0.0018	-0.0017

C. Comparing the Degree of Social Immobility among Those Whose Father Was Born in Europe or America and Those Whose Father Was Born in Israel, Using the Gini Social Immobility Index

Components of the Difference	**Gini Social Immobility Index**	**Bootstrap Standard Deviation**	**Conf. Int. Lower Bound (95%)**	**Conf. Int. Upper Bound (95%)**
Social immobility among those whose father born in Europe or America (G_1)	0.12357	0.0029	0.1189	0.1283
Social immobility among those whose father was born in Israel (G_2)	0.14696	0.0034	0.1414	0.1525
Difference (G_2 - G_1)	0.02338	0.0005	0.0225	0.0243
"Net" difference in social immobility	0.04875	0.0011	0.0469	0.0506
Difference due to difference in the margins of the social mobility matrix	- 0.02537	0.0006	-0.0263	-0.0244
Difference due to differences in the in the "educational composition" of the parents' generation	- 0.02041	0.0005	-0.0212	-0.0196
Difference due to differences in the income distribution of the children's generation	- 0.00496	0.0001	-0.0051	-0.0048

TABLE 3. MEASURING INEQUALITY IN CIRCUMSTANCES

A. On the Basis of Data on the Social Mobility from Fathers to Daughters in France, Using the Theil Index

Income Class	Theil Index of Income Class and Overall Theil Index of Inequality in Circumstances	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)	Share of Each Income Class in Overall Population Analyzed	Contribution of Each Income Class to Overall Theil Index of Inequality in Circumstances	Contribution (in percentage) of Each Income Class to Overall Theil Index of Inequality in Circumstances
less than 4,000 FF	0.07427	0.0017	0.0714	0.0771	0.0636	0.00472	0.0711
from 4,000FF to 5,999FF	0.12520	0.0029	0.1204	0.1300	0.1136	0.01423	0.2142
from 6000FF to 7,999FF	0.03017	0.0007	0.0290	0.0313	0.1422	0.00429	0.0646
from 8,000FF to 9,999FF	0.04145	0.0009	0.0399	0.0430	0.1425	0.00516	0.0777
from 10,000FF to 11,999FF	0.05072	0.0012	0.0488	0.0526	0.1083	0.00549	0.0827
from 12,000FF to 14,999FF	0.03633	0.0008	0.0349	0.0377	0.1534	0.00557	0.0839
from 15,000FF to 19,999FF	0.05963	0.0014	0.0573	0.0619	0.1312	0.00782	0.1178
20,000FF or more	0.11720	0.0027	0.1127	0.1217	0.1632	0.01913	0.2880
All Income Classes Together	0.06642	0.0015	0.0639	0.0689		0.06642	

B. On the Basis of Data on the Social Mobility from Fathers to Daughters in France, Using the Gini Index

Income Class	Gini Index of Income Class and Overall Gini Index of Inequality in Circumstances	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)	Share of Each Income Class in Overall Population Analyzed	Contribution of Each Income Class to Overall Gini Index of Inequality in Circumstances	Contribution (in percentage) of Each Income Class to Overall Gini Index of Inequality in Circumstances
less than 4,000 FF	0.1957	0.0045	0.1882	0.2032	0.0636	0.00079	0.0328
from 4,000FF to 5,999FF	0.2653	0.0061	0.2553	0.2753	0.1136	0.00342	0.1420
from 6000FF to 7,999FF	0.1221	0.0028	0.1175	0.1267	0.1422	0.00247	0.1024
from 8,000FF to 9,999FF	0.1461	0.0033	0.1406	0.1516	0.1425	0.00226	0.0939
from 10,000FF to 11,999FF	0.1632	0.0038	0.1569	0.1695	0.1083	0.00191	0.0794
from 12,000FF to 14,999FF	0.1463	0.0034	0.1407	0.1519	0.1534	0.00344	0.1428
from 15,000FF to 19,999FF	0.1848	0.0042	0.1779	0.1917	0.1312	0.00318	0.1320
20,000FF or more	0.2485	0.0058	0.2390	0.2580	0.1632	0.00662	0.2746
All Income Classes Together	0.02411	0.0006	0.0232	0.0250		0.02411	

C. On the Basis of Data on Social Mobility in Israel, among Those Whose Father was Born in Asia or Africa, Using the Gini Index

Income Class	Gini Index of Income Class and Overall Gini Index of Inequality in Circumstances	Bootstrap Standard Deviation	Conf. Int. Lower Bound (95%)	Conf. Int. Upper Bound (95%)	Share of Each Income Class in Overall Population Analyzed	Contribution of Each Income Class to Overall Gini Index of Inequality in Circumstances	Contribution (in percentage) of Each Income Class to Overall Gini Index of Inequality in Circumstances
NIS 2,000 or less	0.2112	0.0049	0.2032	0.2192	0.0435	0.00040	0.0177
NIS 2,001 to 3,000	0.1957	0.0045	0.1883	0.2031	0.0621	0.00075	0.0335
NIS 3,001 to 4,000	0.2758	0.0064	0.2652	0.2864	0.0932	0.00239	0.1062
NIS 4,001 to 5,000	0.3002	0.0070	0.2887	0.3117	0.0745	0.00167	0.0740
NIS 5,001 to 7,000	0.1231	0.0028	0.1184	0.1278	0.1677	0.00346	0.1536
NIS 7,001 to 9,000	0.2413	0.0056	0.2321	0.2505	0.1304	0.00411	0.1822
NIS 9,001 to 12,000	0.2325	0.0054	0.2237	0.2413	0.1366	0.00396	0.1755
NIS 12,001 to 15,000	0.1618	0.0037	0.1557	0.1679	0.1118	0.00302	0.1341
NIS 15,001 to 20,000	0.1784	0.0041	0.1716	0.1852	0.0497	0.00223	0.0990
More than NIS 20,000	0.2205	0.0050	0.2122	0.2288		0.00054	0.0242
All Income Classes Together	0.02253	0.0005	0.0217	0.0234		0.02253	

FIGURE 1. (Gross) Social Immobility Curve versus Curve of Inequality in Circumstances France: Father's Occupation versus Daughter's Income

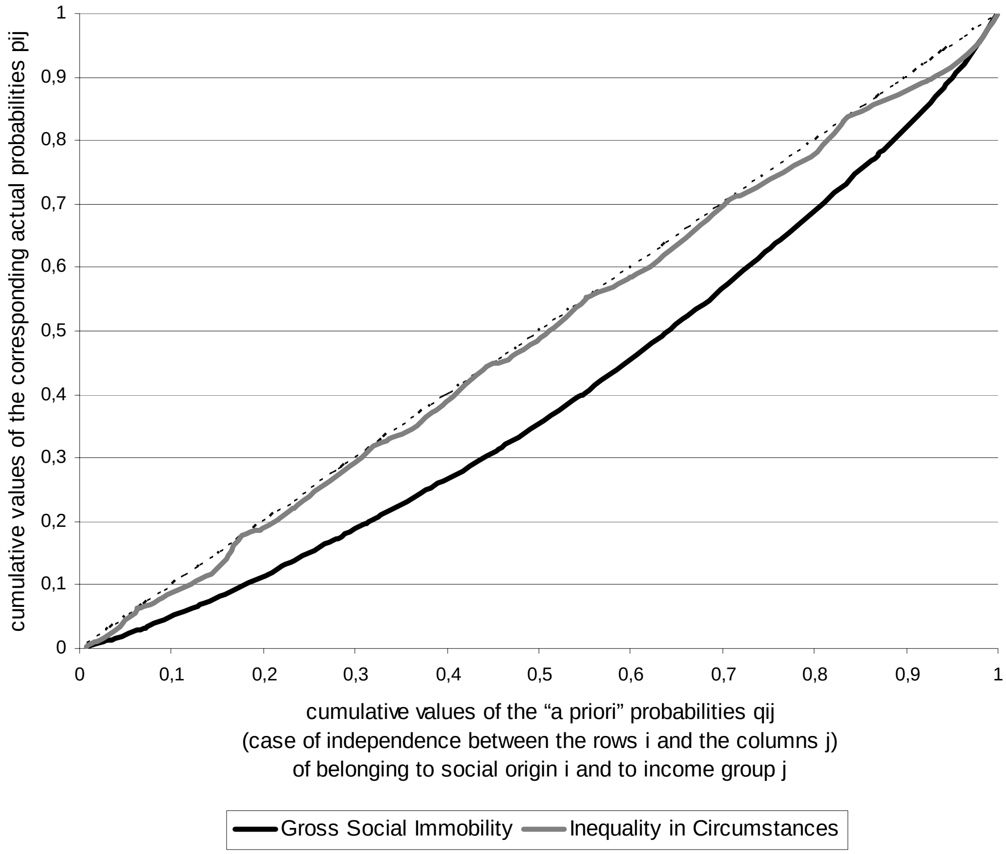
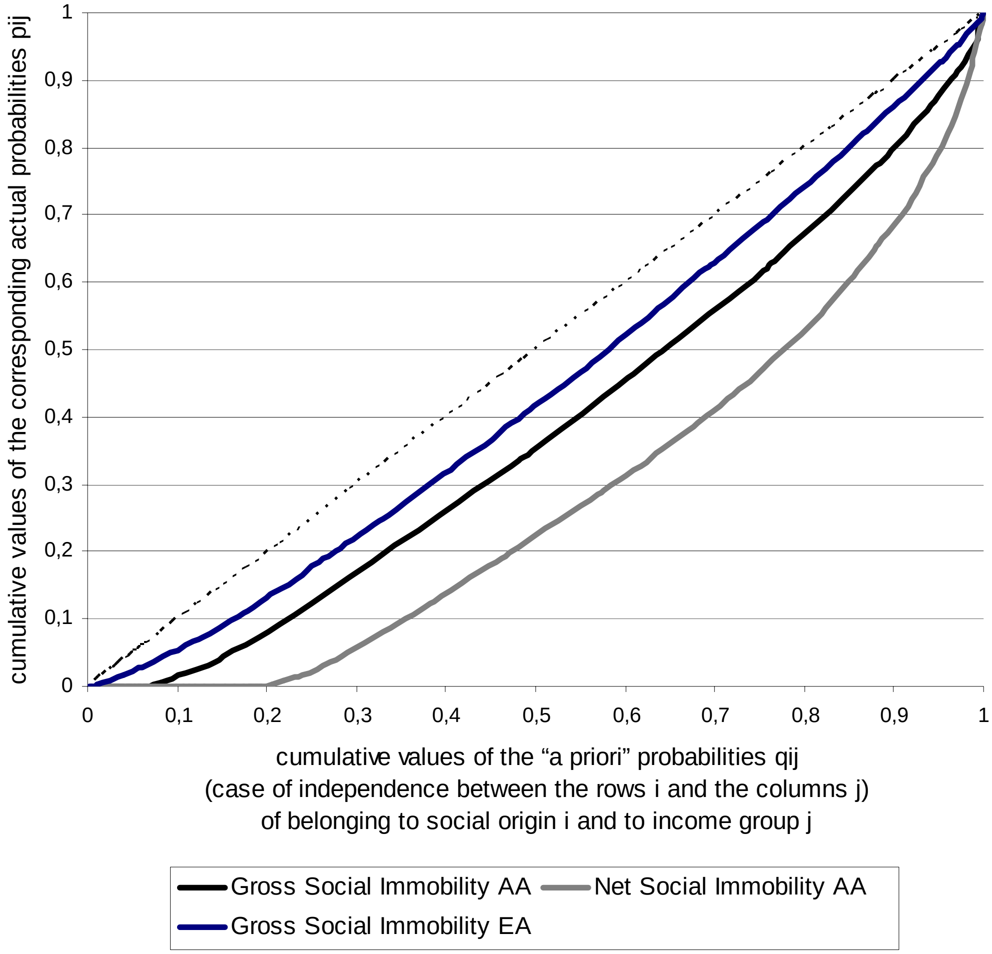


FIGURE 2. Gross and Net Social Immobility Curves Israel: Fathers born in Europe or America (EA) versus Fathers born in Asia or Africa (AA)



APPENDIX A. The Theil Social Immobility Index and the Impact of Two "Margin-Preserving" Transfers

Assume first that $(m_{fh}/(m_{f.}\, m_{.h})) > (m_{fk}/(m_{f.}\, m_{.k}))$ and that $(m_{lk}/(m_{l.}\, m_{.k})) > (m_{lh}/(m_{l.}\, m_{.h}))$. Assume then that two "transfers" of size δ take place, one from m_{fh} to m_{fk}, and one from m_{lk} to m_{lh}. Since the combination of these two transfers implies that there was no change in the marginal shares, the change in the Theil index of immobility may be expressed as:

$$\begin{aligned}
\Delta T_{im} = {} & (m_{f.}\, m_{.h})\{\ln [(m_{f.}\, m_{.h})/(m_{fh}-\delta)] - \ln [(m_{f.}\, m_{.h})/(m_{fh})]\} \\
& + (m_{f.}\, m_{.k})\{\ln [(m_{f.}\, m_{.k})/(m_{fk}+\delta)] - \ln [(m_{f.}\, m_{.k})/(m_{fk})]\} \\
& + (m_{l.}\, m_{.h})\{\ln [(m_{l.}\, m_{.h})/(m_{lh}+\delta)] - \ln [(m_{l.}\, m_{.h})/(m_{lh})]\} \\
& + (m_{l.}\, m_{.k})\{\ln [(m_{l.}\, m_{.k})/(m_{lk}-\delta)] - \ln [(m_{l.}\, m_{.k})/(m_{lk})]\} \\
\leftrightarrow \Delta T_{im} = {} & (m_{f.}\, m_{.h})[\ln (m_{fh}) - \ln (m_{fh}-\delta)] \\
& + (m_{f.}\, m_{.k})[\ln (m_{fk}) - \ln (m_{fk}+\delta)] \\
& + (m_{l.}\, m_{.h})[\ln (m_{lh}) - \ln (m_{lh}+\delta)] \\
& + (m_{l.}\, m_{.k})[\ln (m_{lk}) - \ln (m_{lk}-\delta)] \qquad \text{(A-1)}
\end{aligned}$$

Let us now express ΔT_{im} in (A) as $\Delta T1 + \Delta T2$ with

$$\Delta T1 = m_{f.}\ \{(m_{.h})[\ln (m_{fh}) - \ln (m_{fh}-\delta)] + (m_{.k})[\ln (m_{fk}) - \ln (m_{fk}+\delta)]\} \qquad \text{(A-2)}$$

$$\Delta T2 = m_{l.}\ \{(m_{.h})[\ln (m_{lh}) - \ln (m_{lh}+\delta)] + (m_{.k})[\ln (m_{lk}) - \ln (m_{lk}-\delta)]\} \qquad \text{(A-3)}$$

ΔT_{im} in (A-1) will certainly be negative since the Theil index decreases when a "progressive transfer" is made. As a consequence, the two transfers previously mentioned will increase social mobility.

What does the condition that $\Delta T_{im} = \Delta T1 + \Delta T2 < 0$ imply? Using (A-2) and (A-3) we see that it implies that

$$\begin{aligned}
& m_{f.}\ \{(m_{.h})[\ln (m_{fh}) - \ln (m_{fh}-\delta)] + (m_{.k})[\ln (m_{fk}) - \ln (m_{fk}+\delta)]\} \\
+ \ & m_{l.}\ \{(m_{.h})[\ln (m_{lh}) - \ln (m_{lh}+\delta)] + (m_{.k})[\ln (m_{lk}) - \ln (m_{lk}-\delta)]\} < 0 \qquad \text{(A-4)}
\end{aligned}$$

Let us now call ε_1 the difference [ln (m_{fh}) – ln (m_{fh}-δ)] and ε_2 the difference [ln (m_{fk}+δ) – ln (m_{fk})].

Similarly, let us call β_1 the difference [ln (m_{lh}+ δ) – ln (m_{lh})] and β_2 the difference [ln (m_{lk}) – ln (m_{lk}- δ)].

Condition (A-4) may then be expressed as

$$m_{f.} [m_{.h} \varepsilon_1 - m_{.k} \varepsilon_2] + m_{l.} [- m_{.h} \beta_1 + m_{.k} \beta_2] < 0 \qquad \text{(A-5)}$$

Two possibilities now arise. Either we assume that $[m_{.h} \varepsilon_1 - m_{.k} \varepsilon_2] > 0 \leftrightarrow (m_{.h} / m_{.k}) > (\varepsilon_2 /\varepsilon_1)$, but then (A-5) amounts to assuming that $(m_{f.} / m_{l.}) < [- m_{.h} \beta_1 + m_{.k} \beta_2] / [m_{.h} \varepsilon_1 - m_{.k} \varepsilon_2]$.

But since, by definition, $(m_{f.} / m_{l.}) > 0$, this implies that we assume also that $[- m_{.h} \beta_1 + m_{.k} \beta_2] > 0 \leftrightarrow (m_{.h} / m_{.k}) < (\beta_2 /\beta_1)$

Both conditions then lead to

$$(\varepsilon_2 /\varepsilon_1) < (m_{.h} / m_{.k}) < (\beta_2 /\beta_1) \qquad \text{(A-6)}$$

The other possibility is that we assume that $[m_{.h} \varepsilon_1 - m_{.k} \varepsilon_2] < 0$
$\leftrightarrow (m_{.h} / m_{.k}) < (\varepsilon_2 /\varepsilon_1)$

In such a case, (A-5) amounts to assuming that $(m_{f.} / m_{l.}) > [- m_{.h} \beta_1 + m_{.k} \beta_2] / [m_{.h} \varepsilon_1 - m_{.k} \varepsilon_2]$

But since, by definition, $(m_{f.} / m_{l.}) > 0$, this implies that we assume also that $[- m_{.h} \beta_1 + m_{.k} \beta_2] < 0 \leftrightarrow (m_{.h} / m_{.k}) > (\beta_2 /\beta_1)$

Both conditions then lead to

$$(\beta_2 /\beta_1) < (m_{.h} / m_{.k}) < (\varepsilon_2 /\varepsilon_1) \qquad \text{(A-7)}$$

Remember, however, that we defined ε_1 as [ln (m_{fh}) – ln (m_{fh}-δ)] and ε_2 as [ln (m_{fk}+δ) – ln (m_{fk})]. But [ln (m_{fh}) – ln (m_{fh}-δ)] ≈ (δ/ m_{fh}) and [ln (m_{fk}+δ) – ln (m_{fk})] ≈ (δ/ m_{fk}). Therefore $(\varepsilon_2 /\varepsilon_1) \approx ((\delta/ m_{fk})/ (\delta/ m_{fh})) = (m_{fh})/(m_{fk})$.

Similarly, we defined β_1 as [ln (m_{lh}+ δ) – ln (m_{lh})] and β_2 as [ln (m_{lk}) – ln (m_{lk}- δ)]. But [ln (m_{lh}+ δ) – ln (m_{lh})] ≈ (δ/ m_{lh}) and [ln (m_{lk}) – ln (m_{lk}- δ)] ≈ (δ/ m_{lk}). Therefore $(\beta_2 /\beta_1) \approx ((\delta/ m_{lk})/ (\delta/ m_{lh})) = (m_{lh})/(m_{lk})$

Condition (A-6) may therefore be rewritten as

$$(m_{fh})/(m_{fk}) < (m_{.h} / m_{.k}) < (m_{lh})/(m_{lk}) \quad \text{(A-8)}$$

and condition (A-7) as

$$(m_{lh})/(m_{lk}) < (m_{.h} / m_{.k}) < (m_{fh})/(m_{fk}) \quad \text{(A-9)}$$

Let us now examine what the conditions implied by the two "progressive transfers" previously mentioned.

We first recall that we assumed previously that $(m_{fh}/(m_{f.}\, m_{.h})) > (m_{fk}/(m_{f.}\, m_{.k}))$, which is equivalent to assuming that

$$(m_{fh}/m_{fk}) > (m_{.h}/m_{.k}) \quad \text{(A-10)}$$

We had also assumed that $(m_{lk}/(m_{l.}\, m_{.k})) > (m_{lh}/(m_{l.}\, m_{.h}))$, which is equivalent to assuming that

$$(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) \quad \text{(A-11)}$$

Combining (A-10) and (A-11) we conclude that the two simultaneous transfers imply that

$$(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) \quad \text{(A-12)}$$

Comparing (A-8) and (A-9) on one hand with (A-12) on the other, we conclude that only condition (A-9) [which is also condition (A-12)] is possible.

Let us now see whether we need to assume anything concerning the ordering of m_{lh}, m_{lk} ,$m_{.h}$, $m_{.k}$, m_{fh} and m_{fk}.

Case 1: Assume that $m_{fh} > m_{fk}$ and that $m_{lh} < m_{lk}$.

This means, as we saw previously, that $(\varepsilon_2/\varepsilon_1) > 1$ and $(\beta_2/\beta_1) < 1$. But, as stressed before,

$$(\varepsilon_2/\varepsilon_1) > 1 \leftrightarrow (m_{fh})/(m_{fk}) > 1 \quad \text{(A-13)}$$

and

$$(\beta_2/\beta_1) < 1 \leftrightarrow (m_{lh}/m_{lk}) < 1. \tag{A-14}$$

Combining (A-12), (A-13), and (A-14) we conclude that Case 1 amounts to assuming either that

$$(m_{lh}/m_{lk}) < 1 < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) \tag{A-15}$$

or that

$$(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < 1 < (m_{fh}/m_{fk}) \tag{A-16}$$

Case 2: Assume that $m_{fh} > m_{fk}$ and that $m_{lh} > m_{lk}$.

This means, as we saw previously, that $(\varepsilon_2/\varepsilon_1) > 1$ and $(\beta_2/\beta_1) > 1$ and, as a consequence, that

$$(m_{fh})/(m_{fk}) > 1 \quad \text{and} \quad (m_{lh}/m_{lk}) > 1 \tag{A-17}$$

Combining (A-12) and (A-17) we conclude now that Case 2 amounts to assuming that

$$1 < (m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) \tag{A-18}$$

Case 3: Assume that $m_{fh} < m_{fk}$ and that $m_{lh} < m_{lk}$.

This means, as we saw previously, that $(\varepsilon_2/\varepsilon_1) < 1$ and $(\beta_2/\beta_1) < 1$ and, as a consequence, that

$$(m_{fh})/(m_{fk}) < 1 \quad \text{and} \quad (m_{lh}/m_{lk}) < 1 \tag{A-19}$$

Combining (A-12) and (A-19) we conclude now that Case 3 amounts to assuming that

$$(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) < 1 \tag{A-20}$$

Case 4: Assume that $m_{fh} < m_{fk}$ and that $m_{lh} > m_{lk}$.

This means, as we saw previously, that $(\varepsilon_2/\varepsilon_1) < 1$ and $(\beta_2/\beta_1) > 1$ and, as a consequence, that

$$(m_{fh})/(m_{fk}) < 1 \quad \text{and} \quad (m_{lh}/m_{lk}) > 1 \tag{A-21}$$

However, since (A-12) implies that $(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk})$, Case 4 is impossible.

The three other cases therefore leave us with one of the following possibilities:

a) $(m_{lh}/m_{lk}) < 1 < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk})$

b) $(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < 1 < (m_{fh}/m_{fk})$

c) $1 < (m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk})$

d) $(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) < 1$ (A-22)

We may therefore conclude that the only assumption that needs to be made is that

$$(m_{lh}/m_{lk}) < (m_{.h}/(m_{.k}) < (m_{fh}/m_{fk}) \qquad \text{(A-23)}$$

But (A-23) implies that condition (A-9) will always hold and since condition (A-9) implies that $\Delta T_{im} < 0$, we may safely conclude that the sum of two “margin-preserving“ transfers will always lead to a decrease in the value of the Theil immobility index.

APPENDIX B. On the Concept of Shapley Decomposition

The concept of Shapley decomposition is a technique borrowed from game theory but extended to applied economics by Shorrocks (1999) and Sastre and Trannoy (2002).

Assume an indicator (*I*) is a function of three determinants *a,b,c* and is written as *I*= *I*(*a,b,c*). *I* could be an index of inequality, but, more generally, any function of variables, this function being linear or not.

There are obviously 3!=6 ways of ordering these three determinants *a, b,* and *c*: *(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a)*

The idea of the Shapley decomposition is to compute the **marginal** contribution of each of the three determinants a,b,c.

Let us take the case of a. The marginal contribution C(a) of a is computed by comparing the value of I when a≠0 and its value when a=0. However, in each of these two cases (a≠0 and a=o), the other determinants (b and c) may themselves be different from 0 or equal to. Hence, the importance of taking into account all the possible orderings of a, b, and c.

All the following cases therefore have to be taken into account when computing the marginal contribution of determinant a:

1) I(a≠0;b≠0;c≠0) – I(a=0;b≠0;c≠0)
2) I(a≠0;b≠0;c≠0) – I(a=0;b≠0;c≠0)
3) I(a≠0;b=0;c≠0) – I(a=0;b=0;c≠0)
4) I(a≠0;b≠0;c=0) – I(a=0;b≠0;c=0)
5) I(a≠0;b=0;c=0) – I(a=0;b=0;c=0)
6) I(a≠0;b=0;c=0) – I(a=0;b=0;c=0)

It is then easy to conclude that

C(a) = (2/6)[I(a≠0;b≠0;c≠0) – I(a=0;b≠0;c≠0)]
+ (1/6)[I(a≠0;b=0;c≠0) – I(a=0;b=0;c≠0)]
+ (1/6)[I(a≠0;b≠0;c=0) – I(a=0;b≠0;c=0)]
+ (2/6)[I(a≠0;b=0;c=0) – I(a=0;b=0;c=0)]

One can similarly compute the marginal contributions C(b) and C(c).

Assuming finally that $I(a=0;b=0;c=0) = 0$, one can easily prove that $C(a) + C(b) + C(c) = I(a\neq0;b\neq0;c\neq0)$.

APPENDIX C. Decomposing Variations over Time in the Extent of Social Mobility

In the previous section we have shown how the "move" from the matrix $\{m_{ij}\}$ to the matrix $\{v_{ij}\}$ really included two stages: one in which the margins were changed and one in which the internal structure of the matrix was modified. Let $\Delta SM = SM(v) - SM(m)$ refer to the overall variation in the extent of social mobility. ΔSM may also be expressed as $\Delta SM = f(\Delta m, \Delta is)$, where Δm and Δis refer, respectively, to the variation in the margins of the matrix and in its internal structure. Using Shapley's decomposition (see Shorrocks 1999; Sastre and Trannoy 2002) the contribution $C_{\Delta ma}$ of a change in the margins to the overall variation ΔSM may be written as:

$$C_{\Delta m} = (1/2)\ f(\Delta ma) + (1/2)\ [f(\Delta ma, \Delta is) - f(\Delta is)] \qquad \text{(C-1)}$$

Similarly, the contribution $C_{\Delta is}$ of the change in the internal structure of the matrix to the overall variation ΔSM will be expressed as:

$$C_{\Delta is} = (1/2)\ f(\Delta is) + (1/2)\ [f(\Delta ma, \Delta is) - f(\Delta ma)] \qquad \text{(C-2)}$$

It is easy to observe that $C_{\Delta ma} + C_{\Delta is} = \Delta SM$

Using the definitions of the matrices $\{s_{ij}\}$and $\{w_{ij}\}$given in Section 2.B.1, we derive that the contributions $C_{\Delta ma}$ and $C_{\Delta is}$ may be also written as

$$C_{\Delta ma}=(1/2)[SM(s)–SM(m)]+(1/2)\{[SM(v) – SM(m)]–[SM(w) – SM(m)]\} \qquad \text{(C-3)}$$

$$\Leftrightarrow C_{\Delta ma} = (1/2)\ \{[SM(s) – SM(m)] + [SM(v) – SM(w)]\} \qquad \text{(C-4)}$$

$$C_{\Delta is}=(1/2)[SM(w)–SM(m)]+(1/2)\{[SM(v)–SM(m)]–[SM(s)– SM(m)]\} \qquad \text{(C-5)}$$

$$\Leftrightarrow C_{\Delta is}=(1/2)\{[SM(w) – SM(m)] + [SM(v) – SM(s)]\} \qquad \text{(C-6)}$$

Applying the idea of a Nested Shapley decomposition (see Sastre and Trannoy 2002), we can now further decompose the contribution $C_{\Delta ma}$. In Section 2.B.1 we had defined the matrix $\{r_{ij}\}$ as that which is obtained by multiplying the cells (i,j) of $\{m_{ij}\}$ by the ratios $(v_{i.}/m_{i.})$ where $v_{i.}$ and $m_{i.}$ are, respectively, the horizontal margins of the matrices $\{m_{ij}\}$and $\{v_{ij}\}$. Let us now call

$\{n_{ij}\}$ a matrix which is obtained by multiplying the cells (i,j) of $\{m_{ij}\}$ by the ratios $(v_{.j}/m_{.j})$ where $v_{.j}$ and $m_{.j}$ are, respectively, the vertical margins of the matrices $\{m_{ij}\}$and $\{v_{ij}\}$. Using again the idea of Shapley decomposition applied this time to the difference D_{ma1} defined as

$$D_{ma1} = [SM(s) – SM(m)] \qquad (C-7)$$

we may write that

$$D_{ma1} = g(\Delta h, \Delta t) \qquad (C-8)$$

where Δh and Δt refer, respectively, to the changes in the horizontal and vertical margins. The contributions $C_{\Delta h1}$ and $C_{\Delta t1}$ to the difference D_{ma1} may be then defined as

$$C_{\Delta h1} = (1/2)\ g(\Delta h) + (1/2)\{[g(\Delta h, \Delta t)] – g(\Delta t)\} \qquad (C-9)$$

$$C_{\Delta t1} = (1/2)\ g(\Delta t) + (1/2)\{[g(\Delta h, \Delta t)] – g(\Delta h)\} \qquad (C-10)$$

Using the definitions of $\{r_{ij}\}$and $\{n_{ij}\}$ given previously, these contributions $C_{\Delta h1}$ and $C_{\Delta t1}$ may be also expressed as

$$C_{\Delta h1} = (1/2)\ [SM(r) - SM(m)] + (1/2)\{[SM(s) – SM(m)] – [SM(n) – SM(m)]\}$$

$$\Leftrightarrow C_{\Delta h1}=(1/2)\{[SM(r)-SM(m)]+[SM(s)–SM(n)]\} \qquad (C-11)$$

Similarly

$$C_{\Delta t1}=(1/2)[SM(n)-SM(m)]+(1/2)\{[SM(s)–SM(m)]–[SM(r)–SM(m)]\} \qquad (C-12)$$

$$\Leftrightarrow C_{\Delta t1}=(1/2)\{[SM(n)-SM(m)]+ [SM(s) – SM(r)]\} \qquad (C-13)$$

Let us now decompose in a similar way the difference D_{ma2} defined as

$$D_{ma2} = [SM(v)– SM(w)] \qquad (C-14)$$

We now define two additional matrices, $\{c_{ij}\}$ and $\{f_{ij}\}$, defined as follows. The elements c_{ij} of the matrix $\{c_{ij}\}$ are obtained by multiplying the elements v_{ij} of the matrix $\{v_{ij}\}$ by the ratios $(w_{i.}/v_{i.})$, where $w_{i.}$ and $v_{i.}$ refer to the horizontal margins of the matrices $\{w_{ij}\}$ and $\{v_{ij}\}$. Similarly, the elements f_{ij} of the matrix $\{f_{ij}\}$ are obtained by multiplying the elements v_{ij} of the matrix $\{v_{ij}\}$ by the ratios $(w_{.j}/v_{.j})$ where $w_{.j}$ and $v_{.j}$ refer to the vertical margins of the matrices $\{w_{ij}\}$and $\{v_{ij}\}$. We may therefore define the contributions $C_{\Delta h2}$ and $C_{\Delta t2}$ to the difference D_{ma2} as

$$C_{\Delta h2}=(1/2)[SM(v)-SM(c)]+(1/2)\{[SM(v)–SM(w)]–[SM(v)–SM(f)]\} \quad \text{(C-15)}$$

$$\Leftrightarrow C_{\Delta h2}=(1/2)\{[SM(v)-SM(c)]+ [SM(f) – SM(w)]\} \quad \text{(C-16)}$$

Similarly

$$C_{\Delta t2}=(1/2)[SM(v)-SM(f)]+(1/2)\{[SM(v)–SM(w)]–[SM(v)–SM(c)]\} \quad \text{(C-17)}$$

$$\Leftrightarrow C_{\Delta t2}=(1/2)\{[SM(v)-SM(f)]+[SM(c)– SM(w)]\} \quad \text{(C-18)}$$

Combining now (C-4), (C-7), (C-11), (C-13), (C-14), (C-16), and (C-18) we conclude that the contribution $C_{\Delta ma}$ may be written as

$$C_{\Delta ma} = C_h + C_t \quad \text{(C-19)}$$

where

$$C_h=(1/4)\{[(SM(r)-SM(m))+(SM(s)–SM(n))]+[(SM(v)–SM(c))+(SM(f)–SM(w)]\} \quad \text{(C-20)}$$

$$C_t = (1/4)\{[(SM(n)-SM(m))+(SM(s)–SM(r))]+[(SM(v)-SM(f))+(SM(c)–SM(w))]\} \quad \text{(C-21)}$$

Combining (C-20) and (C-21) we observe that, as expected,

$$C_h+C_t=(1/2)\{[(SM(s)–SM(m)]+[SM(v)–P(w)]\} = C_{\Delta ma} \quad \text{(C-22)}$$

APPENDIX D. The Bootstrap Principle [based on Bradley and Tibshirani (1993)].

The problem solved by bootstrapping can be formulated as follows. We have a random sample $X = (x_1,\ldots x_n)$, obtained from an unknown probability distribution A and we want to estimate a parameter (e.g., the index) $\theta = t(A)$ on the basis of X.

We calculate an estimation of $\hat{\theta} = s(X)$ using X; then the problem is to know how accurate this estimate is. Bootstrapping technique is based on resampling with replacement.

Each bootstrap sample X^* is an independent random sample of size n from the empirical distribution followed by X (that we call Â). To each bootstrap sample corresponds a bootstrap estimation of $\hat{\theta}$: $\hat{\theta}^* = s(X^*)$ that is the results of applying to X^* the same function $s(\ )$ which has been applied to X. The bootstrap algorithm for estimating the standard error and the confidence intervals can be summarised by the following steps:

1) Select B independent bootstrap samples $X_1^*, X_2^*, \ldots, X_B^*$, each consisting of n data values drawn with replacement from X (a good rule of thumb is B = 1000).
2) Evaluate the bootstrap replication corresponding to each bootstrap sample $\hat{\theta}^*(b) = s(X_b^*)$ with $b=1, 2, \ldots, B$
3) Estimate the standard error using the formula:

$$s\hat{e}_B = \left\{ \sum_{b=1}^{B} \left[\hat{\theta}^*(b) - \sum_{b=1}^{B} \hat{\theta}^*(b)/B \right]^2 /(B-1) \right\}^{1/2}$$

and the confidence intervals as: $\left[\hat{\theta} - z^{(1-\alpha)} s\hat{e}_B;\ \hat{\theta} + z^{(\alpha)} s\hat{e}_B\right]$ where z^{α} is the α^{th} percentile of the standardized normal distribution.

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