Working Paper No. 461

Wage Growth and the Measurement of Social Security's Financial Condition

by

Jagadeesh Gokhale

July 2006

Jagadeesh Gokhale is senior fellow at the Cato Institute. The author thanks Andrew Biggs for extensive discussions on all aspects of this paper. Andrew also provided SSASIM-model-based simulation results that are reported in this paper. But for institutional constraints, Andrew would be a co-author on this paper. The author also thanks Alan Auerbach, Michael Boskin, Jeffery Brown, Edward DeMarco, Stephen Goss, Liqun Liu, Joyce Manchester, Donald Marron, Scott Muller, David Pattison, Rudolph Penner, Andrew Rettenmaier, Thomas Saving, Kent Smetters and seminar participants at the Social Security Administration and the Levy Institute at Bard College for helpful comments. The views expressed herein are the author's and do not necessarily represent the views of the Cato Institute.

The Levy Economics Institute
P.O. Box 5000
Annandale-on-Hudson, NY 12504-5000
http://www.levy.org

Abstract

Government spending on the elderly is projected to increase rapidly as the U.S. population becomes older, and many policymakers and budget analysts are concerned about the continued viability of entitlement programs such as Social Security. The Social Security trustees' economic growth projections receive considerable attention because many people believe that higher growth would significantly improve the program's actuarial balance (that is, reduce its actuarial deficit). This belief is validated by Social Security trustees' calculations that show larger 75-year actuarial balances under faster assumed real wage growth rates. Since 2003 the trustees have reported the program's actuarial balance measured in perpetuity. But they do not provide sensitivity analysis that examines the impact of various assumptions on the infinite-term actuarial balance.

This paper shows analytically that faster wage growth may *reduce* Social Security's infinite-term actuarial balance if the ratio of workers to retirees continues to decline rapidly beyond the 75th year. This result holds even if the decline in that ratio ceases after just two decades beyond the 75th year. The paper reports stylized calculations of the impact of real wage growth and demographic change–including time-varying rates of change based on official projections for the U.S. economy–on Social Security's actuarial balance in a multi-period setting. Finally, the Social Security and Accounts Simulator (SSASIM) actuarial model of Social Security financing is used to estimate the degree to which increased wage growth could negatively affect the system's infinite-term actuarial balance.

These results raise questions about the conventional wisdom that holds that improved wage growth would affect Social Security's financing, and how a widely used measure of Social Security's financing captures those effects.

Keywords: Social Security, wage growth, demographic change, infinite horizon, actuarial balance, sustainability

JEL Classifications: H55, E62

1. INTRODUCTION

It is often argued in both policy circles and the popular media that faster economic growth could significantly reduce Social Security's long-term funding imbalance.[1] If, as many argue, Social Security trustees' projections for economic growth are unduly pessimistic, policymakers may ignore calls for policies to reform the system in the belief that faster economic growth will "bail us out." However, Social Security's financial status is normally analyzed under a truncated horizon of 75 years. Does the positive association of faster economic growth with improvement in the system's actuarial balance survive under longer horizons? If not—that is, if faster economic growth fails to improve or even worsens Social Security's actuarial balance over very long horizons—failure to enact reforms to make the system sustainable would be a more serious lapse than many policymakers and budget analysts realize.

The current Social Security benefit formula indexes workers' earnings through age 60 for wage growth when calculating their average indexed monthly wage (AIME), which is the basis for computing Social Security benefits.[2] Benefits are calculated at retirement by applying a progressive formula to the AIME, so that a larger fraction of pre-retirement earnings are replaced by Social Security benefits for low wage workers compared to higher earners. Post-retirement, benefits are increased annually with the Consumer Price Index to maintain their purchasing power. Each worker cohort's retirement benefits—as calculated when its members retire—reflect that cohort's higher labor productivity and wages during its lifetime compared to that of the immediately preceding cohort. Thus, average benefits for succeeding cohorts of retirees will tend to rise at the rate of average wages. And for each cohort, once the benefit level is established, its purchasing power is maintained by allowing the dollar amount to grow at the rate of general price inflation.

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[1] See, for instance, Gordon (2003); Baker (1996); Weller and Rassell (2000); Baker and Weisbrot (1999); and Hall (2005). For contrasting views, see Penner (2003); Davis (2000); Biggs (2000). Note that this paper does not comment on the appropriateness of the wage growth projections made by the Social Security trustees or other agencies. For analysis of the trustees' projections, see the 1999 and 2003 Technical Panel reports, as well as General Accounting Office (2000).

[2] This is done to place past earnings on par with current ones by inflating the former at the rate of nominal wage growth during the intervening years. This accounts for both economy-wide general price inflation and real wage growth that occurred during those years. Disability benefits are calculated in a similar way, though with adjustments for decreased time in the labor force.

Actuarial balance is the most prominent of a number of measures that the Social Security trustees use to assess the program's long-term finances. It equals the present value of the system's annual net income expressed as a percentage of payrolls over the measurement period.[3] As described by the trustees,

> "…actuarial balance is a measure of the program's financial status for the 75-year valuation period as a whole. It is essentially the difference between income and cost of the program expressed as a percentage of taxable payrolls over the valuation period. This single number summarizes the adequacy of program financing for the period."

While the trustees have traditionally measured actuarial balance over 25, 50, and 75 years, the 75-year measure receives the most attention in policy debates. The 2005 Trustees' Report projects a 75-year actuarial deficit of 1.92 percent of taxable payrolls. This deficit has a commonly applied policy interpretation:

> "When the actuarial balance is negative, the actuarial deficit can be interpreted as the percentage that would have to be added to the current law income rate in each of the next 75 years, or subtracted from the cost rate in each year, to bring the funds into actuarial balance." (Board of Trustees 2005)

Under this interpretation, the actuarial deficit indicates the size of an immediate and permanent payroll tax increase—1.92 percentage points, from 12.40 percent to 14.32 percent of wages up to the taxable limit—that would be sufficient to restore the program to actuarial balance over 75 years, though not necessarily thereafter.[4]

Given how past wages enter into the calculation of Social Security benefits, it is easy to understand why many people believe that faster economic growth would improve the system's

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[3] More specifically, the trustees measure actuarial balance over a measurement period as the net value of the initial trust fund balance, the present value of income, the present value of costs, and the present value of scheduled benefits in the final year of the measurement period. This last amount is to satisfy the requirement that the ratio of trust fund assets to benefit payments in the final year equal 100 percent.

[4] The trustees' 2006 report was not available at the time of writing this paper. It should be noted that actuarial balance is not the sole "finite horizon" measure used to assess Social Security's finances. For example, the Social Security trustees also report measures of "close actuarial balance." See the Board of Trustees (2005), p. 60. The Social Security Administration's Office of the Chief Actuary have suggested additional measures, including cash balances and trust fund ratios in specific years and the direction of both at the close of a measurement period. See Goss (1999) and Chaplain and Wade (2005).

financial outlook. Benefits paid to current retirees are indexed only to inflation, rather than to nominal wage growth (which generally exceeds inflation by the growth rate of real labor productivity). Thus, faster growth in real productivity and wages would cause an immediate increase in the tax base and, therefore, in revenues, but would increase benefit payments only after a delay as working generations that experienced faster wage growth retire and claim benefits in the future. If the increase in wage growth were permanent, the annual cost rate – projected benefits as a percent of the projected tax base through the calculation horizon – would permanently decline relative to a lower wage growth scenario. Thus, cash balances relative to the payroll base would improve in every following year.

The Trustees' Annual Report for 2005 shows that over a 75-year horizon, this improvement in annual balances would carry over to an improvement in Social Security's actuarial balance. Assuming an increase in real wage growth from a baseline of 1.1 percent per year to 1.6 percent, the 75-year actuarial balance would improve by 0.53 percentage points, from a deficit of 1.92 percent of payroll to a deficit 1.39 percent. This analysis lends credence to the widely shared view that faster economic growth would significantly *reduce* Social Security's projected actuarial deficit. Moreover, this view is reinforced under other standard measures of Social Security's finances, such as annual balance ratios, the cross-over date (when non-interest receipts begin falling short of program outlays), the date of trust-fund exhaustion, and summarized actuarial balances calculated over truncated horizons of 25, 50, or 75 years. This is labeled as the "traditional" view.[5]

In recent years, Social Security analysts have increasingly focused on very long term financing with the policy goal being solvency that can be sustained well beyond the traditional 75-year scoring period – often termed "sustainable solvency." [6] The trustees note that:

> "Even a 75-year period is not long enough to provide a complete picture of Social Security's financial condition…. Overemphasis on summary measures for a 75-year period can lead to incorrect perceptions and to policy prescriptions that do not move toward a sustainable system. Thus, careful consideration of the trends in annual deficits and unfunded obligations toward the end of the 75-year period is important. In order to provide a more complete description of Social Security's very long-run financial condition, this

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[5] Although economic growth is a broader concept than real wage growth, the two are generally understood to occur concomitantly, at least in public debates about Social Security financing.

[6] See 1994-96 Advisory Council; 1999 Technical Panel; 2003 Technical Panel; and 2005 Trustees' Report.

report also includes summary measures for a time period that extends to the infinite horizon." (Board of Trustees 2005)

Proponents of longer-term measures argue that focusing on 75-year solvency alone can distort policy decisions; the 1999 Technical Panel, for instance, argued that "When reformers aim only for 75-year balance, … they usually end up in a situation where their reforms only last a year before being shown out of 75-year balance again." For that reason, analysts have begun to calculate the Social Security program's finances beyond 75 years.[7] Beginning with the 2003 Report, Social Security's trustees have published data on system financing measured over the infinite term. The main rationale for the infinite horizon measure is that it gives the fullest view of the total assets and obligations of the Social Security program. The Department of the Treasury notes that:

> "…a 75-year projection is incomplete. For example, when calculating unfunded obligations, a 75-year horizon includes revenue from some future workers but only a fraction of their future benefits. In order to provide a complete estimate of the long-run unfunded obligations of the programs, estimates should be extended to the infinite horizon." (Department of the Treasury 2004)[8]

Since then, measures of very long term financing, both for social insurance programs and the federal budget in general, have gained increasing prominence in policy discussions.[9]

Calculations of long-term financing measures suggest that the traditional view may be an artifact of calculating Social Security's actuarial balance under a truncated projection horizon of

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[7] For example, see Gokhale and Smetters (2003, 2006b), and Auerbach, Gale, and Orzag (2004). In infinite horizon calculations, cash flows are projected into the future until the present value sums of future dollar flows (benefits, taxes, the tax base, and so on) become stable (asymptote to a finite value).

[8] See Also Gokhale and Smetters 2006a

[9] Greenspan (2003) discusses the advantages of the related approach of accrual accounting for Social Security; Walker (2003) discussed "one possible approach would be to calculate the estimated discounted present value of major spending and tax proposals as a supplement to, not a substitute for, the CBO's current 10-year cash flow projections." Senator Joseph Lieberman (D-Conn.) has introduced legislation (S. 1915, The Honest Government Accounting Act of 2004) that would calculate 75-year and infinite horizon net present value measures on a government-wide basis. The Social Security Advisory Board's 2003 Technical Panel on Assumptions and Methods praised the trustees' inclusion of measures of system financing in perpetuity and recommended that they be given greater prominence in the Report.

75-years. In particular, such limited-horizon measures reduce the effect of a projected decline in the worker-to-beneficiary ratio over the very long-term. Under perpetuity calculations, the conclusion that faster wage growth improves Social Security's actuarial balance could be reversed when the decline in the worker-to-beneficiary ratio is assumed to continue beyond the next 75 years. This result arises because a declining worker-to-beneficiary ratio magnifies the future impact of faster wage growth on Social Security's cost rate and widens the gap between the present value of its outlays and revenues to yield a *larger* actuarial deficit.

Exploring the sensitivity of Social Security's actuarial balance to individual economic assumptions involves examining its response to changes in one (economic or demographic) parameter at a time. However, altering the real wage growth assumption raises the question of "model consistency." Faster real wage growth could not occur isolated from changes in other relevant economic variables. For example, faster wage growth may be the result of technological progress, which increases the productivity of both capital and labor, and could be associated with higher interest rates. In that case, Social Security's (risk-free) rate of interest could also be higher with accompanying effects on the system's actuarial balance.

If the increase in the government's interest rate associated with faster real wage growth were sufficiently large, bigger future Social Security outlays would receive a smaller weight in present-value calculations, potentially confirming the traditional view. However, because interaction of faster wage growth with a declining worker-to-beneficiary ratio worsens Social Security's long-term actuarial balance under a constant discount rate, such a worsening may persist despite a simultaneous increase in the government's interest rate—up to a limit. With the actuarial balance calculation calibrated to U.S. demographics and real wage growth, it can be shown that faster wage growth would generate smaller actuarial balances for a range of government interest rates.

This paper analyzes the effect of increased economic growth on Social Security solvency measured in perpetuity.[10] Using a simple stylized model of pay-as-you-go Social Security, it can be shown analytically that faster wage growth would reduce the system's actuarial balance if the

[10] Admittedly, a change in economic growth over the long term would be associated with changes in other variables involved in measuring Social Security's financial status—such as wage growth, demographic change, capital returns, and discount rates. This paper does not attempt to capture the interrelationships between these variables in a dynamic general equilibrium setting. Rather, it is limited to examining the impact of higher productivity and real

ratio of workers to beneficiaries declines sufficiently rapidly. These results are examined under a variety of demographic and interest rate assumptions. Next, the SSASIM actuarial model is used to show that such a decline in Social Security's infinite-term actuarial balance is plausible under demographic and economic conditions projected for the United States, even though the program's 75-year actuarial balance would improve under those conditions.

The paper closes with a discussion of the results' meaning for Social Security financing and for the measures of solvency commonly applied to it. The discussion reconciles a seeming contradiction where wage growth improves cash balance ratios in each year but can worsen actuarial balance over the period.

2. A SIMPLE MODEL OF SOCIAL SECURITY FINANCING

This section builds a stylized model of a pay-as-you-go Social Security program. The initial specification is deliberately simplified for the purpose of better communicating the core insights, with increasing complexity and realism added as the model is developed.

Consider a program in which each beneficiary is paid a benefit equal to a constant percentage of the average wage in that year. The actuarial balance (*AB*) for such a program is defined as the present value of taxes minus the present value of benefits, expressed as a percentage of the present value of future payrolls.

$$AB = \frac{PVTaxes - PVBenefits}{PVPayroll} \ . \qquad (1)$$

This is the familiar equation in which the summarized cost rate is subtracted from the summarized income rate.[11]

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wage growth on "static" measures of the program's financial condition that are traditionally used by the Social Security Administration and the program's trustees.

[11] Actuarial balance, as measured by the Social Security trustees, also includes trust fund assets and a requirement that the final year trust fund balance be equal to 100 percent of outlays in that year. To keep the derivations as simple as possible, the formulation of the actuarial balance [equation (1)] in the text assumes those amounts to be zero.

Measured in perpetuity, the present value of taxes can be expressed as:

$$PVTaxes = \tau w_0 \sum_{t=0}^{\infty} N_t G^t R^{-t} \ , \qquad (2)$$

where τ = the payroll tax rate; w_0 = the average wage at time zero; N_t = the population of workers at time t; G = a growth factor ($1+g$), where g equals the annual real wage growth rate; and R = an interest factor ($1+r$), where r equals the government annual interest rate. The present value of benefits equals:

$$PVBenefits = w_0 \beta_0^{-1} \rho \sum_{t=0}^{\infty} N_t G^t B^{-t} R^{-t} \ , \qquad (3)$$

where ρ = a constant replacement rate of the average current wage; β_0 = the worker-to-beneficiary ratio at time zero; and B = a factor $(1-b)$ where b equals the annual rate of decline in the worker-to-beneficiary ratio. The present value of payrolls can be expressed as

$$PVPayrolls = w_0 \sum_{t=0}^{\infty} N_t G^t R^{-t} \ . \qquad (4)$$

Equation (3) shows that the present value of total benefits paid at time t is a function of a constant replacement rate, the initial values of wages, (the inverse of) the worker-to-beneficiary ratio, and changes in the worker population, wages, worker-to-beneficiary ratio, and accumulated interest between time zero and time t.

Note that the values of G^t and B^{-t} would be greater than 1 so long as real wages are rising ($g>0$) and the worker-to-beneficiary ratio is declining ($b>0$); the value of R^{-t} would be less than 1 so long as the real interest rate is positive ($r>0$). Also note that equation (3) assumes that current benefits are a function of *current* wages. That is, there is no lag between realizing higher wages and higher Social Security benefits. This relationship would hold if Social Security benefits were indexed to wages throughout a retiree's lifetime. Although this is not true for Social Security in reality, examining its implications is helpful for developing intuition about results when this assumption is dropped.

The variables in equation (3) affect *PVBenefits* in the following ways: a higher value of g means that wages would be higher in each future period. Because benefits depend on contemporaneous wages by assumption, *PVBenefits* would be larger. Note that if g were larger, each term under the summation in equation (3) and, hence, the entire summation term, would also be larger. The same is true for *PVTaxes* in equation (2). Furthermore, if the t^{th} term in

PVBenefits increases by x percent as a result of an increase in *g*, so would the t^{th} term in *PVTaxes*. Both taxes and benefits would, therefore, increase in the same proportion under a higher value of *g*.

Likewise, if the worker-to-beneficiary ratio declines (that is, if *b* were larger), there would be more beneficiaries per worker in the future, implying a larger *PVBenefits* relative to *PVTaxes* at each given value of *g.* In contrast, increases in the real interest rate (*r*) means that future benefit payments, taxes, and wages are all discounted more heavily—implying proportionate reductions in *PVBenefits, PVTaxes,* and *PVPayrolls*. These relationships are stated as:

Proposition 1:

Assuming 1) that the replacement rate is constant and 2) that current benefits depend on current wages:

i) An increase in real wage growth (g) leads to proportionate increases in PVBenefits, PVTaxes and PVPayrolls;

ii) An increase in the real interest rate (r) leads to proportionate reductions in PVBenefits, PVTaxes and PVPayrolls; and

iii) A faster decline in the worker-to-beneficiary ratio (increase in b), increases PVBenefits relative to both PVTaxes and PVPayrolls.

Using equations (2), (3) and (4), the actuarial balance defined in equation (1) can be expressed as:

$$AB = \frac{\tau N_0 w_0 \sum_{t=0}^{\infty} G^t R^{-t} - N_0 w_0 \rho \beta_0^{-1} \sum_{t=0}^{\infty} B^{-t} G^t R^{-t}}{N_0 w_0 \sum_{t=0}^{\infty} G^t R^{-t}}.$$

Assuming, for simplicity, that the total worker population remains constant over time at N_0, the expression for AB can be expressed as:

$$AB = \tau - \frac{\rho \beta_0^{-1} \sum_{t=0}^{\infty} B^{-t} G^t R^{-t}}{\sum_{t=0}^{\infty} G^t R^{-t}} = \tau - \Omega. \qquad (5)$$

Equation (5) says that the actuarial balance is equal to the tax rate minus the summarized cost rate (Ω), where both revenues and costs are expressed as percentages of payrolls. The assumption of a constant worker population but a declining worker-to-beneficiary ratio obviously implies a growing total population.

Next, the impact of faster wage growth on the actuarial balance is explored under alternative parametric assumptions, progressively making the model more realistic.

Case A. Constant Worker-to-Beneficiary Ratio: This case assumes $b=0$, which implies that the age structure of the population remains constant over time. Then, $B^{-t}=1$ for all future periods t, eliminating it from equation (5) and allowing a simplified expression for the actuarial balance:

$$AB = \tau - \rho\beta_0^{-1}. \qquad (6)$$

Equation (6) is intuitively easy to understand: For a system that receives τ cents per worker to be balanced, that amount must be sufficient to pay benefits to the number of beneficiaries per worker (β_0^{-1}).[12] Note that the compound wage growth term G^t is also eliminated from the expression for AB, implying that in this simplified model (a change in) wage growth does not influence the actuarial balance.

Proposition 2:

With an unchanging population structure (b=0) and with current benefits being proportional to current wages, the Social Security system's actuarial balance is unchanged in response to a change in the rate of real wage growth.

In this simplified setting, a Social Security system that is initially in (out of) balance will remain in (out of) balance to the same degree, regardless of the rate of real wage growth.

Case B: Declining Worker-to-Beneficiary Ratio: Now consider the case where $b>0$—that is, where the worker-to-beneficiary ratio declines over time. First, all other things equal, this will reduce the actuarial balance of the system. With $b>0$, B^{-t} [$=1/(1-b)^t$] must be larger than 1.[13]

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[12] For instance, if the replacement rate were 32 percent and the worker-to-retiree ratio were 2, the tax rate required for a zero actuarial balance would be 16 (32×2^{-1}).

[13] For instance, if b were 2% and t were 5 years, then β^{-t} would be equal to $1/(1 -.02)^5$, or 1.11.

Compared to the cost rate under Case A, a positive *b* increases the numerator in the second term of equation (5) and makes the system's costs as a percentage of payrolls larger, thereby reducing actuarial balance.

Proposition 3:

Other things equal, a faster rate of decline, b, in the worker-to-beneficiary ratio is associated with a smaller actuarial balance.

Moreover, when the worker-to-beneficiary ratio is declining (that is, when $b>0$), the actuarial balance is not neutral with regard to changes in wage growth (*g*) because $B^{-t}>1$ in each future period *t*. In this case, a larger value of *g* causes a disproportionately large increase in the numerator of the second term in equation (5) compared to its denominator causing a change in the actuarial balance.

Proposition 4:

When the worker-retiree ratio is declining (b>0), increased economic growth reduces the actuarial balance.

Although the proof of Proposition 4 is intuitively clear (as described above), a formal proof is provided in Appendix A. Essentially, if the population of retirees is growing, the population of workers is constant, and retirement benefits are determined by current wages, faster wage growth would cause benefit outlays to grow faster than payrolls.

Figure 1 illustrates Propositions 3 and 4 by calculating actuarial balance for a range of values of the parameters *b* and *g*. The system is calibrated to have a zero actuarial balance in perpetuity when annual wage growth is 1 percent and the rate of annual decline in the worker-to-beneficiary ratio is 0.3 percent, roughly the long-term rate projected for the current Social Security program.[14] From this base, the rate of decline of the worker-to-beneficiary ratio is varied in steps (from a low of zero percent to a high of 0.5 percent) and the rate of real wage growth is varied (from zero percent through 2 percent).

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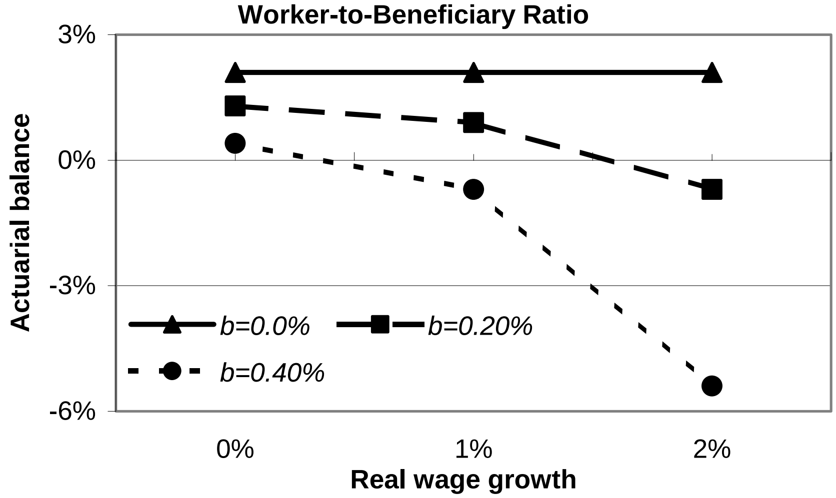
[14] The steady-state decline at the end of the 75-year period is roughly 0.24 percent annually; the rate of decline in the early part of the 75-year period, which has a disproportionate value in actuarial balance calculations, is significantly higher.

Figure 1: Effect of Faster Real Wage Growth on Actuarial Balance Under A Constant Versus A Declining Worker-to-Beneficiary Ratio



When the worker-to-beneficiary ratio is stable (*b=0*), changing the assumed rate of real wage growth has no effect on the actuarial balance. Figure 1 shows that, consistent with Proposition 3, *AB* is smaller at each given level of *g* when $b>0$. In addition, consistent with Proposition 4, when $b>0$, an increases in *g* reduces *AB,* whereas a reduction in *g* increases *AB*. Furthermore, *AB* becomes more sensitive to changes in *g* at larger values of *b*. Thus, in a pure pay-as-you-go program in which benefits are based on current average wages, increased economic growth reduces actuarial balance calculated in perpetuity so long as the worker-retiree ratio is declining.

Case C: Benefits Dependent on Current Wages and Wages Lagged One Period: Equation (3) for *PVBenefits* bears an important distinction from the current Social Security program in that it pays benefits as a percentage of *current* average wages alone, whereas the Social Security program's benefits depend upon *past*, or lagged, wages. As a result, an immediate increase in wages, and thus tax revenues, would not lead to an immediate increase in benefits. This lag in translating wage growth to benefit growth underlies the common belief that system financing unequivocally improves in response to faster economic growth.

Equation (3) is re-written below to express the current benefit as an equally weighted function of current wages and wages 1 period ago. This makes benefits at time *t* a function of wages at time *t* and *t-1*.

$$PVBenefits = \frac{1}{2} w_0 \rho \beta_0^{-1} \sum_{t=0}^{\infty} N_t B^{-t} G^t R^{-t} + \frac{1}{2} w_0 \rho \beta_0^{-1} \sum_{t=0}^{\infty} N_t B^{-t} G^{t-1} R^{-t}$$

$$= \frac{1}{2} w_0 \rho \beta_0^{-1} \sum_{t=0}^{\infty} N_t (1 + G^{-1}) B^{-t} G^t R^{-t} \,. \tag{3a}$$

Given (3a), equation (5) for actuarial balance can be rewritten as:

$$AB = \tau - \frac{\rho \beta_0^{-1} \sum_{t=0}^{\infty} B^{-t} (1 + G^{-1}) G^t R^{-t}}{2 \sum_{t=0}^{\infty} G^t R^{-t}} = \tau - \frac{1}{2}(1 + G^{-1})\Omega \,. \tag{5a}$$

With a stable worker-retiree ratio (*b=0*), equation (6) can be simplified to

$$AB = \tau - \frac{1}{2} \rho \beta_0^{-1} (1 + G^{-1}) \,. \tag{6a}$$

This expression of the actuarial balance clarifies why many people believe that increased economic growth will improve system financing. Because G^{-1} declines as wage growth increases, higher wage growth reduces the cost rate, $(1/2)\rho\beta_0^{-1}(1+G^{-1})$, relative to the revenue rate, τ, and improves the system's financing. This leads to:

Proposition 5:

Assuming 1) a stable worker-to-beneficiary ratio (b=0) and 2) dependence of benefits on lagged wages, faster wage growth reduces the cost rate and improves the system's actuarial balance.

The above discussion clarifies that when benefits are a function of lagged wages, faster wage growth has a differential impact on *PVBenefits* and *PVTaxes*. This becomes clear by comparing equations (5) and (5a).

The obvious next question concerns the impact of faster wage growth (*g*) on the actuarial balance when the worker-to-beneficiary ratio is declining—that is, the sign of the derivative *dAB/dG* when *b>0.*

Appendix B shows that the expression for *dAB/dG* for the case of *b>0* can be written as:

$$\frac{dAB}{dG} = -\omega(1+G^{-1})G^{-1}\Omega\left[Z - \frac{G^{-1}}{1+G^{-1}}\right], \tag{7}$$

where Ω is the summarized cost rate as defined in equation (5) above and *Z* equals the net increase in Ω arising from a change in *G*. As discussed earlier, an increase in *G* would lead to a larger increase in the numerator of the second term of equation (5) compared to the increase in the denominator because each B^{-t} term in the numerator exceeds 1. The term *Z* in equation (7) [defined in equation A6 in Appendix A] equals the net increase in the numerator of the second term in equation (5) compared to the denominator due to an increase in *G*. The term *Z* is a function of *b, Z(b),* with the properties: (i) that Z≥0 when *b*≥0, with equality holding when *b*=0; and (ii) that Z increases monotonically with *b*. Thus, Z in equation (7) captures the impact of the worker-to-retiree ratio on the change in the actuarial balance due to a change in the growth rate (*dAB/dG*).

What is equation 7 telling us? It is simply a combination of Propositions 4 and 5. Proposition 4 revealed that with retirees forming a larger fraction of the population over time, faster wage growth increases Social Security's cost rate and *worsens* the system's actuarial balance. Proposition 5 shows that under dependence of benefits on lagged wages, faster wage growth *improves* the system's actuarial balance. Equation 7 shows that change in the actuarial balance arising from faster wage growth depends on the balance of these opposing forces. Appendix B shows that setting *b*=Z=0 in Equation (7) yields the result of Proposition 5, namely that:

$$\frac{dAB}{dG} = \frac{1}{2}G^{-2}\Omega > 0\,. \tag{8}$$

Equation (7) also clarifies that setting *b>0* (which implies *Z>0*) could change the sign of *dAB/dG* from positive to negative—by flipping the sign of the term in the square brackets. That is, a sufficiently rapid decline in the worker-to-beneficiary ratio could result in Proposition 4's effect dominating that of Proposition 5. That would cause the system's actuarial balance to become smaller (more negative) in response to a change in the wage growth rate, contrary to the popular belief that higher wage growth improves Social Security's finances.

Case D: Benefits Dependent on Wages in Several Earlier Period: In practice, current Social Security benefit outlays are not just a function of wages one period ago but of wages often as many as 40 periods earlier. That's because although the benefits of those retiring today are wage indexed—that is, dependant on current wages—the benefits of today's older retirees are based on wages from several periods ago (that prevailed in their periods of retirement) and now grow only with prices rather than wages. For example, Social Security benefits of those aged 92 today who retired when they were aged 62 are determined by the wage level from 30 periods ago, whereas the benefits of retirees aged 67 today who retired when they were aged 62 depend on the wage level from just 5 years ago. Appendix C shows that if past wages entering the actuarial balance formula are equally weighted, the actuarial balance can be expressed as:

$$AB = -\frac{1}{N+1}\gamma(G)G^{-1}\Omega\left[Z + \frac{(N+1)G^{-(N+1)}}{1-G^{-(N+1)}} - \frac{G^{-1}}{1-G^{-1}}\right] \tag{9}$$

where $\gamma(G)$ summarizes the dependence of benefits on past wages. Again, as Appendix C shows, the basic conclusions of Case C above would be preserved. That is, whether $dAB/dG \lesseqgtr 0$ depends on the balance of two opposing forces. For values of *g* and *b* where the two forces are exactly balanced, $dAB/dG=0$. For other combinations of *g* and *b*, $dAB/dG\neq0$, meaning it would be either negative or positive. This yields:

Proposition 6: *When current benefits are an equally weighted function of wages in the current period and N earlier periods, for each given value of g, there exists a value of* $b^*=b(g)$ *where* $dAB/dG = 0$*, with* $dAB/dG > 0$ *when* $b<b^*$ *and* $dAB/dG < 0$ *when* $b>b^*$*.*[15]

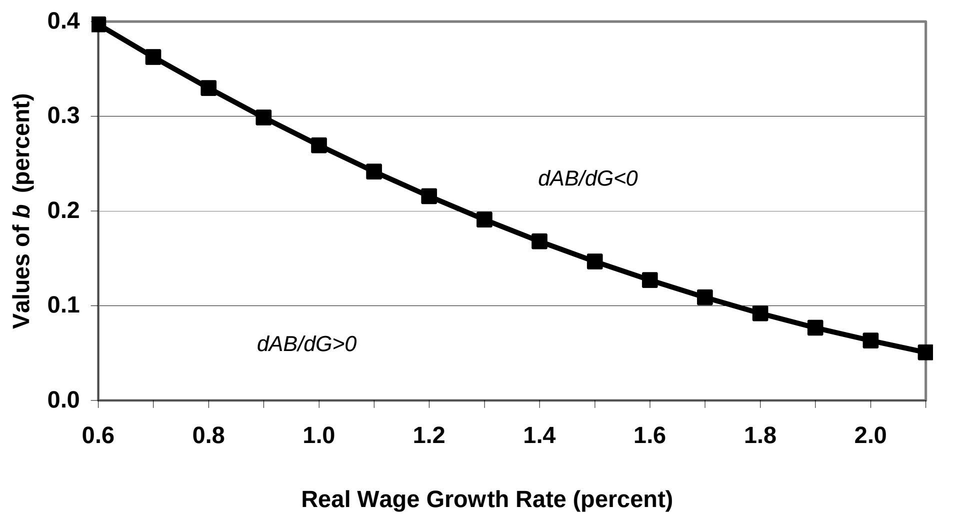
It is obvious that equally weighting past wages in the actuarial balance formula is inappropriate because mortality reduces the size of older cohorts whose benefits are determined by wages further back in the past. Hence, actuarial balance should be calculated using declining weights calibrated to the age distribution of cohort sizes over time. Applying smaller rather than equal weights to wage levels further back in the past implies that the force of Proposition 5 (whereby actuarial balance improves in response to faster wage growth) diminishes relative to that of Proposition 4 in determining the change in actuarial balances with respect to a change in

[15] To keep the development of these propositions simple, current and past wages are weighted equally (see Appendix C). However, sub-section E below shows the results obtained from assuming declining population weights for wage terms occurring earlier in the past.

real wage growth. Because a larger share of total benefits would be paid to relatively younger retirees, faster wage growth would result in larger benefit outlays more quickly. Consequently, the combinations of *g* and *b* values at which $dAB/dG = 0$ would be different compared to the case of equal weighting.

Figure 2 shows locus (that is, combinations of the wage growth rate *g*, and the rate of decline in the worker-to-beneficiary ratio, *b(g)*) for which $dAB/dG = 0$. The calculations assume: τ (payroll tax rate)=12.4%; w_0 (initial real wage)=1; $N_t=N_0$ (population of workers at time *t*)=1; *r* (interest rate)=3%; ρ (benefit replacement rate)=35%; β_0 (initial worker-to-beneficiary ratio)=3.33; and N (the number of past wage periods that enter into the benefit formula)=35.

Figure 2: Locus of *dAB/dG=0* When Current Benefits Depend on Current Wages And Wages In 35 Past Periods -- Declining Weights



In Figure 2, the locus is calculated under the assumption of declining weights for wages further back in the past. The weights are calculated based on population shares of those aged 65 and older that would arise under age-specific conditional mortality rates for those aged 65 and older.[16] The derivative of actuarial balance with respect to *G*, *dAB/dG*, is negative for combinations of *b* and *g* that lie in the north-east direction relative to the locus. That is, higher wage growth would reduce actuarial balance under these circumstances. For wage growth rates

[16] Mortality rates provided by the Social Security Administration are used in calculating the weights.

approximating current rates in the United States – about 1 percent per year – values of $b^*=b(g^*)$ are very small—about 0.2 percent—making it quite likely that $dAB/dG<0$ when b values are calibrated to U.S. demographics.

Case E: Calibration To U.S. Demographic Projections: Figure 3 shows projected values of the worker-to-beneficiary ratio for the United States.[17] It shows that the ratio is expected to decline sharply during the next three decades followed by a much more gradual decline after the baby-boom generation transitions into retirement and passes away.

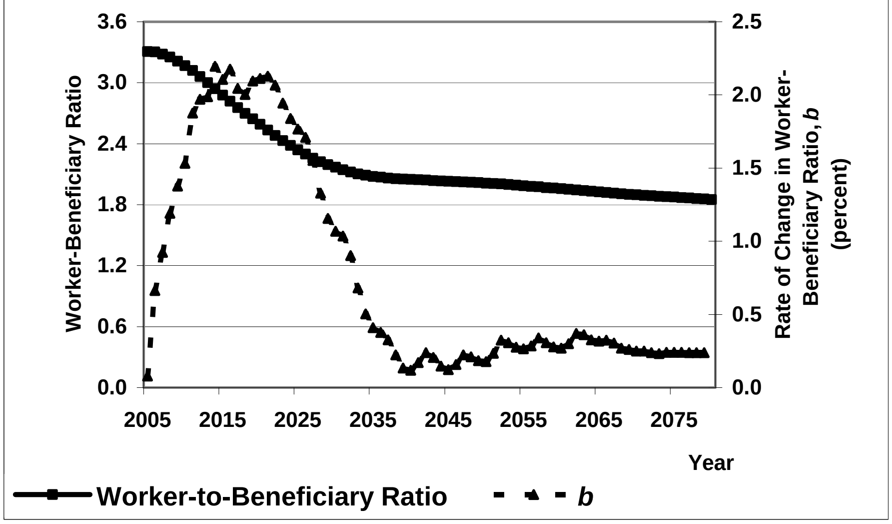


Figure 3: Projected Worker-Beneficiary Ratio and its Rate of Change (*b*) in the United States

Figure 3 also shows the corresponding projected time-varying rate of decline (b) in the worker-to-beneficiary ratio. The values of b are generally quite large compared to the values in Figure 2 where $dAB/dG=0$ when real wage growth equals 1 percent.

Note that while the worker-to-beneficiary ratio is projected to decline, this decline would take place alongside a *growing* projected population in the United States. Figure 4 shows the Social Security Administration's projection of the population of workers and that of workers plus retirees, both normalized to their population sizes in 2005. It indicates that a projected decline in the worker-to-beneficiary ratio does not involve a stagnant worker population as assumed earlier. Rather, both populations are projected to grow in absolute size in the United States. A declining

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[17] All demographic projections are taken from the Social Security Administration. See http://www.ssa.gov/OACT/TR/TR05/lrIndex.html (noted as of January 6, 2005).

worker-to-beneficiary ratio means just that the fraction of the total (and growing) population that would be in the workforce is expected to decline over the next 75 years.

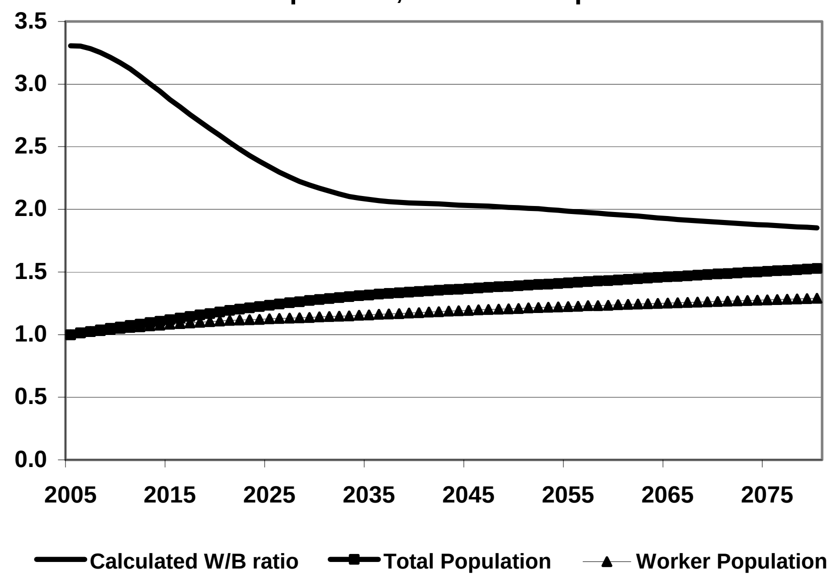


Figure 4: Projected U.S. Worker-Beneficiary Ratio, Worker Population, and Total Populations

3. RESPONSE OF ACTUARIAL BALANCE UNDER FULL CALIBRATION TO U.S. DEMOGRAPHICS

Main Results

Incorporating U.S. demographic projections into the actuarial balance calculation and assuming that the rate of decline in the worker-to-beneficiary ratio beyond the year 2080 remains constant at its 2080 value yields values for dAB/dG of 0.29 when g=1.1 percent and −1.75 when g=1.6 percent. The values of AB at those two values of g are −3.2 percent and −4.1 percent respectively.[18] That is, although at g=1.1 percent the immediate marginal contribution of faster growth is positive, the marginal contribution becomes negative rapidly as g is increased and cumulatively results in a smaller (more negative) actuarial balance when g=1.6 percent.

[18] This stylized model of Social Security financing excludes many details of the actual Social Security program, including the income taxation of benefits, scheduled increases in the normal retirement age, survivor and disability benefits, and actuarial reductions for early retirement. Its actuarial balance estimate should not, therefore, be expected to closely approximate the official estimate of the Social Security's Board of Trustees based on much more detailed calculations. The estimate of a 3.2 percent actuarial deficit under Social Security's intermediate growth and interest rate assumptions appears quite reasonable in comparison with the official estimate of 3.5 percent.

As Table 1 shows, restricting actuarial balance calculations to just 75 years would suggest the opposite conclusion: A larger value of real wage growth (g) produces an

Table 1: Actuarial Balance and Change in Response to Change in Real Wage Growth and Discount Rates					
Discount Rate (%)	Projection Horizon	Real Wage Growth Rate (%)	Actuarial Balance, AB (%)	dAB/dG	Local Elasticity, $\|\varepsilon\|$
2.7	∞	**1.1**	-4.0	-0.582	0.023
2.7	∞	**1.6**	-5.9	-5.657	0.327
2.7	**75**	**1.1**	-1.6	1.783	0.028
2.7	**75**	**1.6**	1.2	1.669	0.021
3.0	∞	**1.1**	-3.2	0.292	0.009
3.0	∞	**1.6**	-4.1	-1.748	0.070
3.0	**75**	**1.1**	-1.5	1.761	0.026
3.0	**75**	**1.6**	-1.1	1.645	0.018
3.3	∞	**1.1**	-2.7	0.749	0.020
3.3	∞	**1.6**	-3.1	-0.314	0.010
3.3	**75**	**1.1**	-1.3	1.742	0.023
3.3	**75**	**1.6**	-1.0	1.624	0.016

(algebraically) larger actuarial balance and positive values of dAB / dG. For example, using the baseline discount rate of 3 percent and real wage growth rate of 1.1 percent, the 75-year horizon yields an actuarial balance of just −1.5 percent, a much smaller deficit than the −3.2 percent obtained under the calculation in perpetuity. In addition, by increasing the growth rate to 1.6 percent per year, the 75-year actuarial balance becomes algebraically larger (less negative): −1.1 percent.

Sensivity Analysis: Discount Rate

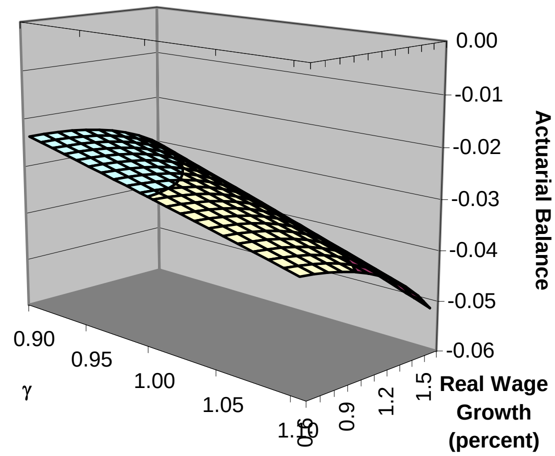
Table 1 also shows that under perpetuity calculations, the (negative) response of actuarial balance to increases in wage growth rates is very large when present values are calculated using smaller discount rates. This is as expected because smaller discount rates increase the weight on dollar flows in the distant future relative to weights on dollar flows in the immediate future, making future benefit obligations larger in present value relative to earlier payroll tax-payments. The opposite result holds when the assumed discount rate is larger – as Table 1 shows.

Sensitivity Analysis: Rate of Decline in b

The next step is to investigate the impact on actuarial balance of a slightly faster or slower decline in the U.S. worker-to-beneficiary ratio when the calculation horizon is infinite. Figure 5 shows the actuarial balance for different values of a parameter, γ (gamma), applied to the time-varying values of *b* shown in Figure 3. For example, γ=0.9 would imply a slower decline in the worker-to-beneficiary ratio over time whereas γ=1.1 would imply a faster rate of decline in that ratio. Figure 5 shows the values of *AB* for values of γ ranging from 0.9 to 1.1 and values of wage growth (*g*) between 0.6 percent and 1.6 percent (that is, values of G ranging from 1.006 to 1.016). Figure 5 shows that at all levels of wage growth within this range, the actuarial balance is smaller (more negative) when γ is increased (the worker-to-beneficiary ratio declines faster). Moreover, Figure 5 shows that for each rate of decline in the worker-to-beneficiary ratio, there is a rate of productivity/wage growth at which the actuarial balance is maximized. At γ=1, the rate of real wage growth that maximizes the actuarial balance is much smaller, only around 0.5 percent—closer to that under the Social Security trustees' "high-cost" assumptions. Under calculations in perpetuity, increasing real wage growth would, according to the figure, reduce Social Security's actuarial balance given projected demographic changes in the United States.[19]

[19] Again, remember that measurement of Social Security's finances is conducted under a "static" framework (see footnote 10).

Figure 5: Actuarial Balance for Alternative Rates of Decline in the Worker-Beneficiary Ratio and Real Wage Growth



4. ACTUARIAL BALANCE AND ANNUAL BALANCES UNDER FASTER WAGE GROWTH – A CONUNDRUM?

The previous section showed that under stylized calculations calibrated to features of the U.S. Social Security system, faster growth would reduce the infinite horizon actuarial balance. Appendix D shows, however, that with benefits dependent on past wages, annual balance ratios (total payroll taxes minus total benefits as a ratio of total payrolls in any given year) would be *larger* in all future years.

This result seemingly creates a policy conundrum: The infinite-horizon actuarial balance is usually interpreted as immediate and permanent payroll tax hike required to balance the system's intertemporal budget constraint. A reduction in the actuarial balance under faster wage growth means that such a tax increase must be larger. However, an increase in each future year's annual balance ratios under faster wage growth implies that the "pay-as-you-go" tax rate increase that must be levied in each future year would be smaller. It is tempting to conclude, therefore, that the pay-as-you-go approach to resolving Social Security's shortfalls would be better than pre-funding.

Appendix D shows that the actuarial balance can be expressed as a product of an "annual-balance effect" and a "weighting effect." It shows that equation for the actuarial balance can be expressed as:

$$AB = \sum_{t=0}^{\infty} \frac{Cash\ Balance_t}{Payrolls_t} \frac{PVPayrolls_t}{\sum_{t=0}^{\infty} PVPayroll_t} = \sum_{t=0}^{\infty} \left[\frac{\tau G^t - \rho \beta_0^{-1} \omega \gamma(G) B^{-t} G^t}{G^t} \right] \bullet \left[\frac{G^t R^{-t}}{\sum_{t=0}^{\infty} G^t R^{-t}} \right], \qquad (10)$$

where $\omega\ \gamma(G)$ captures the impact of past wages levels on current benefits, and $G=1+g$, and $B=1-b$, and $R=1+r$ as before (see Section 2).

In equation (10), the first term in square brackets equals the annual balance ratio in period t and the second term in square brackets is the present valued weight applied to year-t's annual balance ratio. According to equation (10), the weighted sum of all future annual balance ratios equals the actuarial balance. Appendix D clarifies that annual balance ratios unambiguously increase in each future year under faster real wage growth.

Figure 6: Annual Balance Ratios Alternative Wage Growth Rates (1.1% and 1.6%) -- Stylized Model Under U.S. Calibration

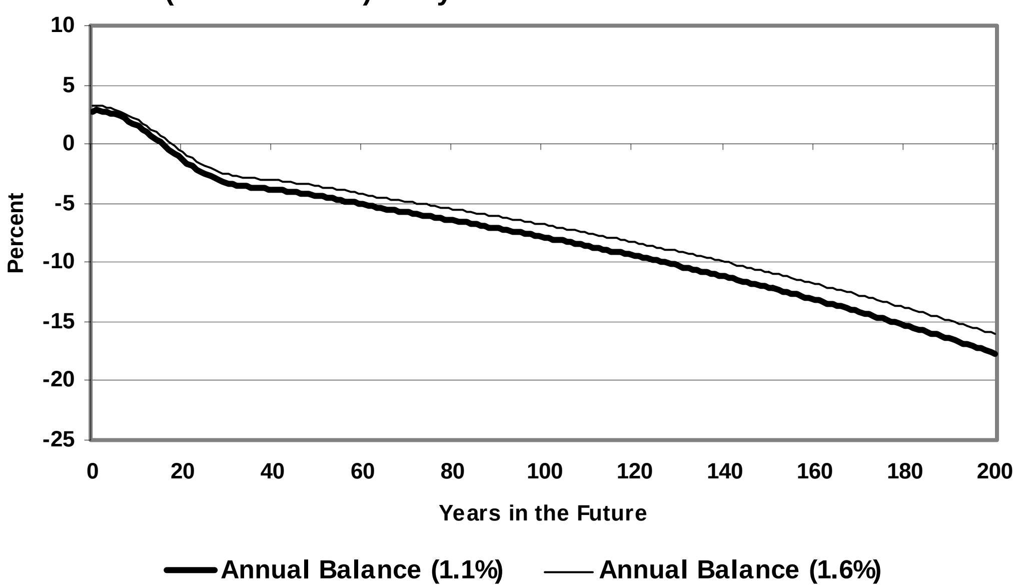


Figure 6 shows unweighted annual balance ratios when the stylized model of the earlier section is calibrated to features of the U.S. Social Security system (corresponding to the cases shown in Table 1 with real wage growth rates of 1.1 and 1.6 percent and a discount rate of 3 percent). It shows that projected annual balance ratios are negative and declining over time. Increasing the real wage growth rate from 1.1 percent to 1.6 percent per year increases annual balance ratios (makes them less negative) in each future year but the ratios still decline over time.

Figure 7: Present Valued Weights Under Alternative Wage Growth Rates (1.1% and 1.6%) -- Stylized Model Under U.S. Calibration.

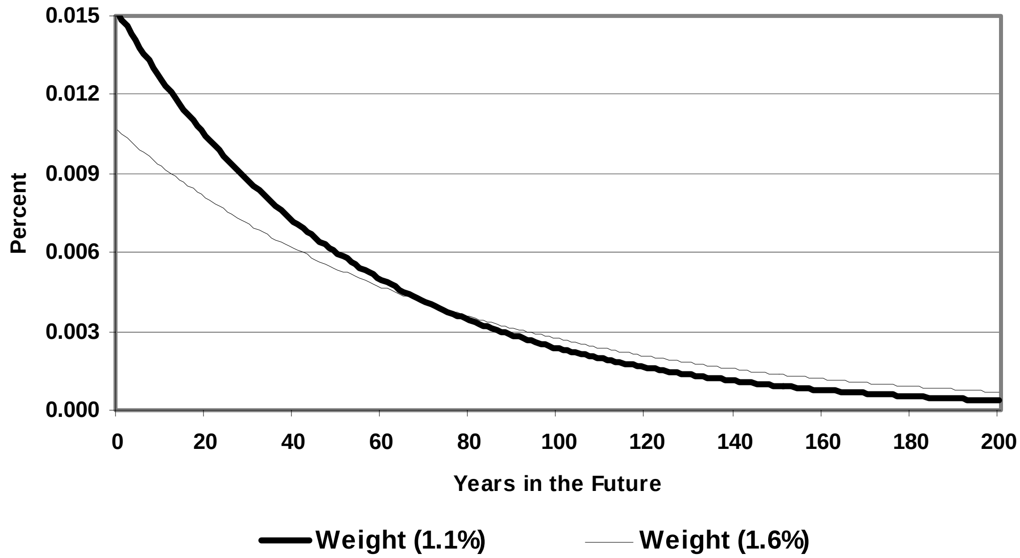


Figure 7 indicates the present valued weights applicable each year according to equation (10) for the same two cases as shown in Figure 6. Figure 7 indicates (as explained in Appendix D) that under faster wage growth, the weights applicable to the earlier annual balance ratios would be smaller. Those applicable to later annual balance ratios would be larger. Under a pay-as-you-go approach to resolving Social Security's future shortfalls, each year's weight can be interpreted as share of future payrolls (in present value terms) that would bear a pay-as-you-go tax rate hike equal to that year's annual balance ratio.

With this information, the policy conundrum mentioned earlier can be resolved. In Table 1, the infinite-horizon actuarial balance was –3.2 percent under a wage growth rate of 1.1 percent and discount rate of 3.0 percent. Although increasing wage growth to 1.6 percent increases unweighted annual cash balance ratios in each future year, it also increases the share of present valued payrolls that would be subjected to pay-as-you-go tax rates larger than 3.2 percentage points and reduces the share of payrolls subject to a pay-as-you-go tax rate of less than 3.2 percentage points. Hence, under faster wage growth, present valued payrolls must, on average, bear a pay-as-you-go tax rate that is larger—4.1 percentage points according to Table 1.

The conundrum mentioned earlier is resolved in the following sense. Although faster growth leads to smaller pay-as-you-go tax *rate hikes* in each future year, the share of future wages that is subject to larger tax pay-as-you-go tax *rates* (compared to the average rate under slower wage growth) increases. The latter (weighting effect) may be sufficiently large under faster wage growth to generate a larger actuarial balance—as appears to be the case when the model is calibrated to features of the U.S. Social Security system. The choice between pay-as-you-go and pre-funding methods for resolving future financial shortfalls is no longer unambiguous because under the former, a larger share of future wages would be subject to higher tax rates when wage growth is faster.

5. MODEL CONSISTENCY IN EVALUATING THE SENSITIVITY OF ACTUARIAL BALANCE

This section considers the issue of "model consistency" when exploring the response of actuarial balance to faster wage growth. The standard criticisms levied against the sensitivity analysis presented in the Social Security trustees' annual reports is that exploring the implications of changing a single factor while holding other inputs constant is inappropriate and the analysis cries out for a general equilibrium framework. For example, faster real wage growth that, perhaps, results from better economic policies, would be accompanied by a different constellation of economic (and, perhaps, demographic) outcomes. Replicating, as is done here, a static approach to analyzing Social Security's finances that is used by most government scoring agencies would be subject to the same criticism; faster wage growth could be accompanied, for example, by higher interest rates as technological shocks increase the productivity of both labor and capital.

There are two responses to these criticisms. First, a general equilibrium framework requires explicit specification of the policies that would be used to close the government's intertemporal budget constraint. The trustees' analysis of Social Security finances imposes no such budget constraint. When the objective is to measure an existing budget gap, general equilibrium modeling is naturally precluded. A standard "budget measure" approach is adopted wherein Social Security is presumed to continue paying scheduled benefits even though revenues are inadequate.

Second, impending demographic change in the United States is likely to increase future capital intensity. A declining pool of workers relative to wealthier retirees would tend to dampen increases in interest rates arising from productivity enhancing technical progress. On the other hand, economic agents may demand higher returns on savings in an environment of higher growth but perhaps also greater economic volatility.

The standard approach to estimating the government's interest rate under uncertainty suggests equating it to the rate of time preference (say, 1 percent per year) plus the product of two items: the inverse of the degree of risk aversion and the standard deviation of productivity growth. However, there is no consensus in the economic growth literature on the size of the appropriate risk aversion parameter.

An inverse relationship between wage growth and Social Security's actuarial balance is supported under a range of interest rates. For example, calculations using the stylized model under Case E using a 3 percent discount rate show that when the real wage growth rate is increased from 0.6 percent per year to 1.6 percent per year, the actuarial balance declines from −3.0 percent to −4.1 percent. However, simultaneously increasing the government's interest rate from 3.0 percent to 3.3 percent would leave the actuarial balance unchanged at −3.0 percent.[20] That implies the actuarial balance could decline when faster wage growth is accompanied with higher interest rates over a limited, but non-trivial, range.

In addition to interest rate uncertainties, the calculations reported earlier assume that the decline in the worker-to-beneficiary ratio would continue indefinitely—implying that Social Security benefits would be financed by workers comprising an ever-smaller fraction of the population.[21]

[20] Under the steady state relationship $r=\rho+(x/\theta)$, where the rate of time preference (ρ) is assumed to equal 1 percent, productivity growth rate alternatives (x) ranging between 0.6 percent and 1.6 percent per year and interest rate alternatives (r) ranging between 2.6 and 3.2 percent per year are consistent with intertemporal elasticities of substitution (θ) between 0.27 and 1.0. These values of θ span the range of values estimated in the economics literature. However, note that this relationship characterizes a steady state, whereas the U.S. economy is undergoing a sizable transition.

[21] Note that a declining share of the worker population in the total population is consistent with both populations growing over time.

Although gradually increasing longevity and a gradual but continuing decline in fertility is not inconceivable for a number of decades beyond the next 75 years, the assumption of declining worker-retiree ratios in perpetuity is difficult to defend.

To explore how crucial this assumption is, the infinite term actuarial balance is calculated under alternative ranges of years beyond the next seventy-five, during which the worker-to-retiree ratio declines, but stops declining thereafter. In other words, assume $b_t=b_{75}$ for $75 \leq t \leq S$ and $b_t=0$ for $t>S$. Table 2 shows changes in the infinite term actuarial balance from increasing wage growth under alternative values of S. It shows that the infinite-term actuarial balance under wage growth of 1.6 percent per year is smaller (implying that the actuarial deficit is larger) than that under wage growth of 1.1 percent per year when the assumption of $b_t=b_{75}$ is maintained for just 20 additional years beyond the next 75 years (S>95). Thus, although the negative impact of higher wage growth on the infinite-term actuarial balance requires the assumption of a continued decline in the worker-to-beneficiary ratio, it does not appear necessary to maintain that assumption for more than a few years beyond the conventional projection horizon of 75 years.

Table 2: Infinite Term Actuarial Balance Under Alternative Horizons for Continued Decline in the Worker-to-Beneficiary Ratio

S	Infinite Term Actuarial Balance Under Alternative Wage Growth Rate Assumptions	
	1.1 percent	1.6 percent
85	-2.51	-2.47
95	-2.61	-2.61
105	-2.70	-2.74

Source: 2005 Social Security Trustees Report, Table VI.D.4

6. SIMULATIONS UNDER A DETAILED MODEL OF SOCIAL SECURITY—SSASIM

These stylized demonstrations of the impact of wage growth on Social Security's actuarial balance capture the essence of the current Social Security program—wherein current benefits are based on past wages—but do not capture the full details of Social Security financing.

This section reports results under the SSASIM (Social Security and Accounts Simulator) model developed and maintained by the Policy Simulation Group at the Social Security Administration. This model was developed during the 1994-1996 Advisory Council on Social Security under contract with a number of organizations, including the Social Security Administration, and has been regularly updated since then.[22] The SSASIM model has two modes of calculating system financing: a cell-based actuarial mode designed to replicate the results from the Social Security Administration's actuaries and a fully microsimulation-based mode similar to that utilized by the Congressional Budget Office. The results reported below were produced using SSASIM's cell-based mode, though simulations using the microsimulation-based mode produce qualitatively similar results. It should be noted, however, that results from the SSASIM model do not constitute official findings from the Social Security Administration's actuaries and official estimates may differ.

A. SSASIM Performance Relative to SSA Estimates

The Social Security Trustees' Report results from sensitivity analyses conducted on a number of demographic and economic factors.

Table 3: Social Security Trustees Sensitivity Analysis of Real Wage Growth on 75-year Actuarial Balance

	Assumed ultimate rate of real wage growth	
Percent of taxable payroll	1.1 percent	1.6 percent
Summarized income rate	13.87	13.74
Summarized cost rate	15.79	15.13
Actuarial balance	-1.92	-1.39

Source: 2005 Social Security Trustees Report, Table VI.D.4

[22] For details, see Holmer (2005); www.polsim.com

These factors are shifted by a pre-set amount from their mid-point projections to examine how increasing or reducing their values affects Social Security's finances over the next 75 years. Table 3 reports the trustees' findings on the sensitivity of the actuarial balance with respect to wage growth: increasing the ultimate rate of real wage growth from 1.1 percent to 1.6 percent increases the 75-year actuarial balance by 0.53 percent of payroll. As expected, system solvency is improved through a decline in the summarized cost rate (the ratio of the present value of benefit outlays plus administrative expenses to the present value of taxable payrolls).[23]

Although the SSASIM model does not use real wage growth as a direct input, changes to assumed rates of productivity growth increase wage growth and impact system financing. The SSASIM baseline productivity growth assumption of 1.6 percent is consistent with that assumed by Social Security's trustees and the model produces a 75-year actuarial deficit of 1.92 percent of taxable payroll, also consistent with the trustees' projections in their 2005 annual report. In the SSASIM model, increasing the assumed rate of annual productivity growth from 1.6 to 2.1 percent (which corresponds to an increase in the real wage growth rate from 1.1 percent to 1.6 percent), produces very similar results. The 75-year actuarial deficit is reduced from 1.92 to 1.42 percent of taxable payroll—an improvement of 0.50 percentage points—quite close to the 0.53 percentage point improvement reported by the trustees.

Since 2003, the annual Social Security Trustees' Report has published estimates of system financing in perpetuity. The 2005 Report estimated the program's actuarial deficit in perpetuity as 3.5 percent of taxable wages, meaning that an immediate and permanent payroll tax increase of 3.5 percentage points would be sufficient to maintain program sustainability under the trustees' intermediate economic and demographic assumptions (Board of Trustees Section IV.B.5) However, the Report does not conduct sensitivity analysis for changes in economic or demographic factors measured over an infinite horizon, as it does for solvency measured over the traditional 75-year horizon.

[23] This improvement emerges despite a *decline* in the summarized income rate (defined as the value of the trust fund plus the present value of tax revenues expressed as a percentage of the present value of taxable payrolls). The decline in the summarized income rate occurs primarily because the initial value of the trust fund, while unchanged in dollar terms, falls relative to the larger present value of taxable payrolls under the high growth scenario. Another minor reason for the decline in the summarized income rate is that part of the program's revenue is derived from income taxes levied on benefit payments. Because benefits increase with a lag, so do those income tax revenues.

When the system's solvency is measured in perpetuity, the SSASIM model produces a revenue shortfall equal to 3.53 percent of the present value of payrolls—very close to the (rounded) 3.5 percent projected by the trustees.[24]

B. SSASIM's Perpetuity Estimate of Sensitivity of Actuarial Balance to Productivity Growth

SSASIM model projections show that increasing the rate of productivity growth from 1.6 percent to 2.1 percent would *increase* Social Security's actuarial deficit from 3.5 percent to 3.7 percent of taxable payroll (that is, reduce it's actuarial balance as defined in equation (5) in Section 2 from –3.5 percent to –3.7 percent). The reason for this is two-fold: economic growth increases costs by *more* than it increases payrolls, and increases income *less* than the increase in payrolls. SSASIM model calculations show that the summarized cost rate increases from 17.25 percent of payroll in the base case to 17.29 percent of payroll in the high-growth scenario (see Table 4). Moreover, the program's income rate declines from 13.72 to 13.59 percent of payroll in response to faster wage growth.[25]

[24] Note that the infinite horizon simulation in SSASIM was conducted using slightly different mortality assumptions than the 75-year forecast. The baseline 75-year projection in SSASIM assumes annual mortality reductions of 0.83 percent, versus 0.71 percent assumed by the trustees, due to differences in how the SSASIM model incorporates changes to mortality. For the infinite horizon simulations, mortality reduction was returned to the 0.71 percent ultimate rate assumed by the trustees. However, using consistent mortality assumptions between the 75-year and infinite horizon simulations does not change the outcome of altering the productivity assumption.

[25] As outlined earlier, much of this is because of the decline in the fixed initial value of the trust fund relative to the larger tax base. SSASIM uses an initial trust fund balance of $1.553 trillion (differing slightly from the $1.501 value in the 2005 Trustees' Report). This amount is equal to 0.48 percent of payroll under the baseline scenario, but only 0.35 percent of payroll when productivity growth is increased from 1.6 percent to 2.1 percent.

Table 4: Impact of Productivity Growth on Infinite Term Income, Cost and Payrolls			
	Productivity growth		
$ trillions present value (percent of payroll)	1.6 percent	2.1 percent	Percent change
Income	$44.52 (13.72%)	$60.30 (13.59%)	35.44%
Cost	$55.97 (17.25%)	$76.80 (17.29%)	37.21%
Payroll	$324.49	$444.00	36.83%
Source: Authors' calculations based on SSASIM model.			

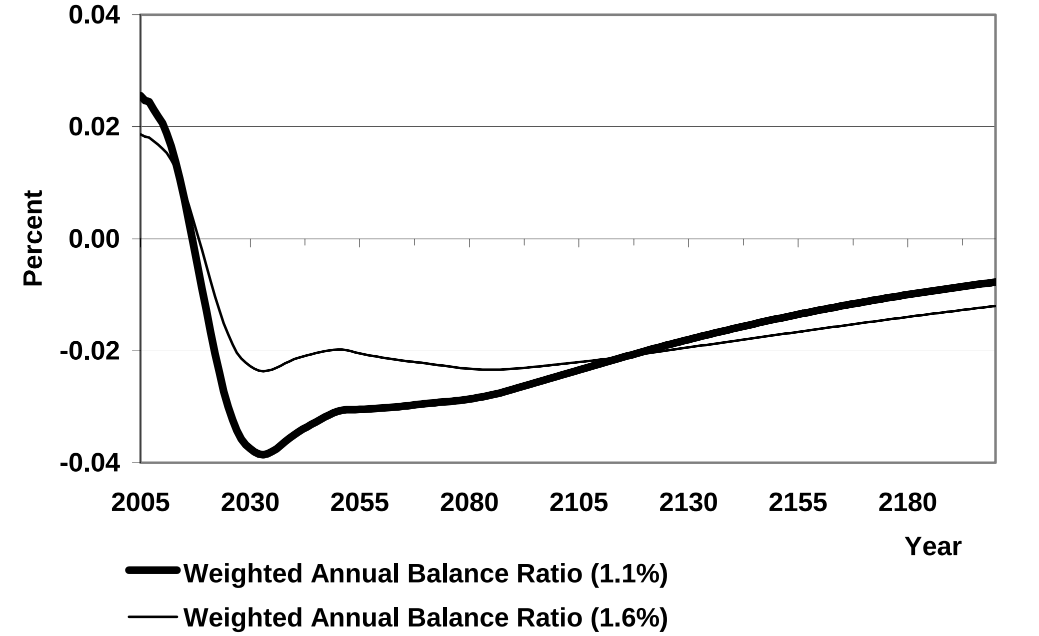
The net impact of these two changes is a decline in the system's actuarial balance from −3.53 to −3.70 percent. Note that the actuarial balance would have declined even if the reduction in the income rate traceable to the existing trust fund were ignored.

The reason for the worsening of the actuarial balance can be traced to the opposing effects identified in Proposition 6 of section 2; a direct actuarial-balance-increasing effect of the lagged dependence of benefits on wages versus the opposite effect due to a decline in the worker-to-beneficiary ratio. The SSASIM model's estimate of a worsening actuarial balance under faster productivity growth suggests that the latter effect dominates the former under an assessment of Social Security's finances in perpetuity.

Figure 8 shows the product of annual balance ratios and weights in each future year (as in equation (10) in Section 4), but utilizes SSASIM (rather than stylized model) results to illustrate the annual cash balances entering the actuarial balance calculation of equation (10).[26]

[26] The actual profiles of annual cash balance ratios are different in Figure 7 compared to Figure 6 because the SSASIM model incorporates tax, benefit, and demographic features relevant for the Social Security program in much greater detail than the stylized model of Section 2.

Figure 8: Weighted Annual Balance Ratios Under Alternative Wage Growth Rates (1.1% and 1.6%) (SSASIM model; *b* >0; Wage-Price Indexing)



Recall that in equation (10), the actuarial balance is expressed as the weighted average of annual balance ratios, with weighting determined by the ratio of each year's discounted payroll to the present value of all payrolls over the measurement period.

Figure 8 illustrates that for roughly the next 115 years, the annual balance effect would dominate the weighting effect (see Appendix D)—that is, annual balance ratios weighted by present valued payroll shares in total present value of payrolls would be smaller under a 1.6 percent wage growth rate than under a 1.1 percent rate. After that period, however, annual improvements in annual cash balance ratios arising from faster wage growth would be insufficient and weighted cash balance ratios would become more negative under g=1.6 percent compared to g=1.1 percent. Hence, the infinite-horizon actuarial balance is smaller under the faster wage growth assumption. Figure 8 also clarifies the seemingly contradictory result that faster wage growth improves actuarial balance over 75 years but reduces it in perpetuity.

7. CONCLUSION

Since 2003, the Social Security trustees have begun to report the system's financial imbalance measured in perpetuity. Unfortunately, the trustees do not report the sensitivity of the perpetuity imbalance measure to alternative economic and demographic assumptions. Using a stylized model of Social Security finances and a detailed Social Security simulation model, both calibrated to the Social Security Administration's intermediate economic and demographic projections, this paper shows that faster real wage growth would substantially worsen Social Security's actuarial balance under the new perpetuity measure. This stands in sharp contrast to the conventional wisdom that faster wage growth would improve Social Security's financial status.

That wisdom has been reinforced under standard measures of Social Security's finances such as annual balance ratios, the cross-over date (when non-interest receipts begin falling short of program outlays), the date of trust-fund exhaustion, and summarized actuarial balances calculated over truncated horizons of 25, 50, or 75 years.[27] The paper's analysis indicates that evaluating Social Security's financial status based on standard measures would be hazardous.

The paper shows that assuming a faster future rate of wage growth would imply that a larger share of total future payrolls must be devoted to pay scheduled benefits. This occurs because although wage growth increases future payrolls and magnifies the financial advantage from the lagged dependence of benefits on wages, the negative impact on Social Security's finances of a persistent decline in the worker-to-beneficiary ratio would dominate—even when the latter is not projected to last for more than a couple of decades beyond the trustees' standard 75-year projection horizon.

The paper also provides a detailed interpretation of why the infinite-term actuarial balance declines under faster wage growth despite the fact that annual balance ratios increase unambiguously in all future years. It shows that under pay-as-you-go tax increases for meeting future Social Security's shortfalls, a larger share of future payrolls would be subject to higher

[27] The Social Security Administration's Office of the Chief Actuary also uses the reduction in the cash deficit in the final year of the 75-year period as a proxy measure for a reform proposal's improvement to the program's cash flows. See, for instance, Chaplain and Wade (2005).

payroll tax rates when wage growth occurs faster. That means the pay-as-you-go approach to resolving Social Security's financial imbalance is not unambiguously preferable to pre-funding.

The paper shows that the decline in Social Security's infinite-term actuarial balance in response to faster real wage growth is preserved under alternative discount rate assumptions. It is also robust to simultaneous increases in wage growth and interest rates over limited ranges of those two variables.

Faster economic growth is obviously desirable because it would help increase living standards and provide additional resources for addressing growing entitlement costs in general. However, given that Social Security's revenues and benefits both depend on wages, faster wage growth would not necessarily improve, and may worsen, Social Security's finances when they are measured using the infinite-term actuarial balance.

APPENDIX A

Proof of Proposition 4

Equation (5) in the text is:

$$AB = \tau - \frac{\rho\beta_0^{-1}\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}} = \tau - \frac{N}{D} = \tau - \Omega\,, \qquad \text{(A5)}$$

where *R=(1+r)*; *G=(1+g); B=(1–b);* and it is assumed that the numerator is well defined—that is *G/BR<1*.

Note: $D = \sum_{t=0}^{\infty} G^{t}R^{-t}$;

$$dD = \sum_{t=0}^{\infty} tG^{t-1}R^{-t}dG = G^{-1}\sum_{t=0}^{\infty} tG^{t}R^{-t}dG\,;$$

$$N = \rho\beta_0^{-1}\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t} \;\; ;$$

and

$$dN = \rho\beta_0^{-1}\sum_{t=0}^{\infty} tB^{-t}G^{t-1}R^{-t}dG = G^{-1}\rho\beta_0^{-1}\sum_{t=0}^{\infty} tB^{-t}G^{t}R^{-t}dG\,.$$

Thus,

$$\frac{dAB}{dG} = -\frac{d\Omega}{dG} = -\frac{DdN - NdD}{D^2} = -\Omega\left[\frac{dN}{N} - \frac{dD}{D}\right]$$

$$= -\Omega\left[\frac{\left[G^{-1}\rho\beta_0^{-1}\sum_{t=0}^{\infty} tB^{-t}G^{t}R^{-t}\right]}{\rho\beta_0^{-1}\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}} - \frac{\left[G^{-1}\sum_{t=0}^{\infty} tG^{t}R^{-t}\right]}{\left[\sum_{t=0}^{\infty} G^{t}R^{-t}\right]}\right]$$

$$= -G^{-1}\Omega\left[\frac{\left[\sum_{t=0}^{\infty} tB^{-t}G^{t}R^{-t}\right]}{\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}} - \frac{\left[\sum_{t=0}^{\infty} tG^{t}R^{-t}\right]}{\left[\sum_{t=0}^{\infty} G^{t}R^{-t}\right]}\right] = -G^{-1}\Omega Z\,. \tag{A6}$$

We know that $G^{-1}>0$ and $\Omega>0$ (cost is positive). In equation (A6), $Z>0$ when $b>0$ (see the Proof A below). That yields the result $[dAB/dG]<0$ when $b>0$.

Proof A:

To prove: $Z = \left[\frac{\left[\sum_{t=0}^{\infty} tB^{-t}G^{t}R^{-t}\right]}{\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}} - \frac{\left[\sum_{t=0}^{\infty} tG^{t}R^{-t}\right]}{\left[\sum_{t=0}^{\infty} G^{t}R^{-t}\right]}\right] \geq 0$ when $b\geq 0$ (with equality holding if $b=0$). That is,

$$\left[\sum_{t=0}^{\infty} tB^{-t}G^{t}R^{-t}\right]\left[\sum_{t=0}^{\infty} G^{t}R^{-t}\right] \geq \left[\sum_{t=0}^{\infty} tG^{t}R^{-t}\right]\left[\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}\right] \tag{A7}$$

Writing $G^{t}R^{-t}=x_t$, equation (A7) can be expressed as

$$\left[\sum_{t=0}^{\infty} tB^{-t}x_t^2\right] + \sum_{i=0}^{\infty} iB^{-i}x_i \sum_{j=0}^{\infty} x_j\Bigg|_{i\neq j} \geq \left[\sum_{t=0}^{\infty} tB^{-t}x_t^2\right] + \sum_{i=0}^{\infty} ix_i \sum_{j=0}^{\infty} B^{-j}x_j\Bigg|_{i\neq j}.$$

Eliminating the first terms on each side of the inequality yields

$$\sum_{i=0}^{\infty} iB^{-i}x_i \sum_{j=0}^{\infty} x_j \Big|_{i\neq j} \geq \sum_{i=0}^{\infty} ix_i \sum_{j=0}^{\infty} B^{-j}x_j \Big|_{i\neq j} .$$

To construct the proof, assume that the opposite (that is, replace ≥ with <) and show that doing so leads to a contradiction:

$$\text{Assume } \sum_{i=0}^{\infty} iB^{-i}x_i \sum_{j=0}^{\infty} x_j \Big|_{i\neq j} < \sum_{i=0}^{\infty} ix_i \sum_{j=0}^{\infty} B^{-j}x_j \Big|_{i\neq j} . \tag{A8}$$

Select any pair of terms in equation (A8) where, $i=n$ and $j=m$, in the first, and $i=m$ and $j=n$ in the second. Without loss of generality, assume $m<n$.

For this pair, the left hand side of (A8) equals $nB^{-n}x_n x_m + mB^{-m}x_m x_n$, and the right hand side equals $nx_n B^{-m}x_m + mx_m B^{-n}x_n$

Expression (A8) implies checking if $nB^{-n}x_n x_m + mB^{-m}x_m x_n < nx_n B^{-m}x_m + mx_m B^{-n}x_n$ for each such pair of terms.

That is, whether $nB^{-n} + mB^{-m} < nB^{-m} + mB^{-n}$;

Multiplying all terms by B^m, check whether $nB^{-(n-m)} + m < n + mB^{-(n-m)}$;

or $B^{-(n-m)}(n-m) < n-m$.

However, given that $B^{-1} \geq 1$ when $b \geq 0$, this inequality cannot be true since $m<n$ by assumption. Because the contradiction applies to all pairs of terms *i, j* [*(n,m)* and *(m,n)* with $m<n$], it applies to equation (A8) in its entirety. Hence, $Z \geq 0$ when $b \geq 0$ (with equality holding when $b=0$).

Moreover, Z is a monotonically increasing function of *b*. This follows from the fact that $B^{-1}=[1/(1-b]$ is a monotonically increasing function of *b*.

APPENDIX B

Proof of Proposition 6

Suppose current benefits are determined by wages in two periods—the current period and 1 period ago. Assume that each period's wages receive the same weight, ω=0.5, in the benefit formula. Equation (A5) would be modified to:

$$AB = \tau - \frac{\rho\beta_0^{-1}\omega(G^0 + G^{-1})\sum_{t=0}^{\infty} B^{-t}G^t R^{-t}}{\sum_{t=0}^{\infty} G^t R^{-t}} = \tau - \omega(1+G^{-1})\Omega = \tau - \omega\left[\frac{1-G^{-2}}{1-G^{-1}}\right]\Omega, \qquad \text{(A9)}$$

where ω=0.5, *G=(1+g),* and *B=(1–b)*. Thus,

$$\frac{dAB}{dG} = -\omega\left[\left[\frac{1-G^{-2}}{1-G^{-1}}\right]\frac{d\Omega}{dG} + \Omega\left[\frac{1-G^{-2}}{1-G^{-1}}\right]\left[\frac{2G^{-3}}{1-G^{-2}} - \frac{G^{-2}}{1-G^{-1}}\right]\right] \qquad \text{(A10)}$$

Using the result from equation (A6) that $\frac{d\Omega}{dG} = G^{-1}\Omega Z$ and simple algebraic manipulations yields

$$\frac{dAB}{dG} = -\omega(1+G^{-1})G^{-1}\Omega\left[Z - \frac{G^{-1}}{1+G^{-1}}\right], \qquad \text{(A11)}$$

where Z is as defined in Proof A above. Note that when *Z=1* when *b=0*. Hence,

$$\lim_{b\to 0}\frac{dAB}{dG} = \omega G^{-2}\Omega > 0\,. \qquad \text{(A12)}$$

When current benefits are a function of current wages and wages one period ago, for each given value of *g* there exists some value, $b^*(g)$, such that $[dAB/dG]\big|_{b=b^*} = 0$. For $b>b^*(g)$, faster wage growth causes the actuarial balance to decline.

APPENDIX C

This Appendix generalizes the case of Appendix B by assuming that the current benefit level is based on the current wage and wages in N past periods. It is assumed that each period's wage receives an equal weight $\omega = 1/(N+1)$. Then, the expression for AB becomes

$$AB = \tau - \frac{\rho\beta_0^{-1}\omega\left(\sum_{i=0}^{N} G^{-i}\right)\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}} = \tau - \omega\left(\sum_{i=0}^{N} G^{-i}\right)\Omega = \tau - \omega\gamma(G)\Omega\,, \qquad \text{(A13)}$$

where $\gamma(G) = \left[\frac{1-G^{-(N+1)}}{1-G^{-1}}\right]$.

Then,

$$\frac{dAB}{dG} = -\omega\Omega[\gamma(G)G^{-1}Z + \gamma'(G)]$$

$$= -\omega\gamma(G)G^{-1}\Omega\left[Z + \frac{(N+1)G^{-(N+1)}}{1-G^{-(N+1)}} - \frac{G^{-1}}{1-G^{-1}}\right] \qquad \text{(A14)}$$

Equation (A14) (which is identical to equation (A11) when N=1) shows that a result similar to that of Appendix B holds: When current benefits are a function of current and wages from N earlier periods, there exists a value $b=b^{**}(g)$ for which $[dAB/dG]\big|_{b=b^{**}} = 0$. For $b>b^{**}(g)$, higher growth causes the actuarial balance to decline.

APPENDIX D

Equation (A13) of Appendix C

$$AB = \tau - \frac{\rho\beta_0^{-1}\omega\left(\sum_{i=0}^{N} G^{-i}\right)\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}} \tag{A13}$$

can also be expressed as

$$= \frac{\tau\sum_{t=0}^{\infty} G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}} - \frac{\rho\beta_0^{-1}\omega\left(\sum_{i=0}^{N} G^{-i}\right)\sum_{t=0}^{\infty} B^{-t}G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}}.$$

Letting $\gamma(G) = \left(\sum_{i=0}^{N} G^{-i}\right)$, simple manipulation allows the actuarial balance to be expressed as

$$AB = \sum_{t=0}^{\infty}\left[\frac{\tau G^{t} - \rho\beta_0^{-1}\omega\gamma(G)B^{-t}G^{t}}{G^{t}}\right]\bullet\left[\frac{G^{t}R^{-t}}{\sum_{t=0}^{\infty} G^{t}R^{-t}}\right]. \tag{A15}$$

Equation (A15) (which corresponds to equation (10) in the text) shows that the actuarial balance is a weighted sum of the ratio of annual net cash flows $(\tau G^{t} - \rho\beta_0^{-1}\omega\gamma(G)B^{-t}G^{t})$ to annual payrolls G^{t} (the "annual cash balance ratio"), where the weight equals $G^{t}R^{-t}/\sum_{t=0}^{\infty} G^{t}R^{-t}$. In annual-cash-balance-ratio term, the dependence of benefits on lagged wages is captured in the term $\gamma(G)$ and ω=1/(N+1), with N being number of past years' wages that factor into the benefit determination (Note: *N* depends on the age of the oldest cohort alive relative to the age of retirement).

The first term in equation (A15) represents the annual balance ratio. Rewriting it as $\left[\tau - \rho\beta_0^{-1}\omega\gamma(G)B^{-t}\right]$ clarifies that every future year's annual balance ratio would be unambiguously larger (that is, annual deficits would be smaller) under faster wage growth. That's because $\gamma(G) = \left(\sum_{i=0}^{N} G^{-i} \right)$ would be unambiguously smaller under faster wage growth. Call this the "annual balance effect."

However, another feature of equation (A15) is that faster wage growth implies larger weights on annual balances accruing in the more distant future. Note that the denominator in $G^t R^{-t} / \sum_{t=0}^{\infty} G^t R^{-t}$ also grows larger, but because it's an average over all future years, it grows at a slower rate than the numerator $G^t R^{-t}$ when t is large. Call this the "weighting effect."

Hence, if the out years are deficit years, (a) those deficits will be smaller because of the annual balance effect but (b) will become more important in the present value calculation because of the weighting effect. The net effect on the actuarial balance could be positive or negative. As Proposition 6 in the text shows, whether the actuarial balance is increased or reduced with faster wage growth depends on the rate at which the worker-to-beneficiary ratio declines over time and the way in which benefits depend on past wages.

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