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Ecology, Economics, and Network Dynamics

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ABSTRACT

In a seminal 1972 paper, Robert M. May asked: "Will a Large Complex System Be Stable?" and argued that stability (of a broad class of random linear systems) decreases with increasing complexity, sparking a revolution in our understanding of ecosystem dynamics. Twenty-five years later, May, Levin, and Sugihara translated our understanding of the dynamics of ecological networks to the financial world in a second seminal paper, "Complex Systems: Ecology for Bankers." Just a year later, the US subprime crisis led to a near worldwide "great recession," spread by the world financial network. In the present paper we describe highlights in the development of our present understanding of stability and complexity in network systems, in order to better understand the role of networks in both stabilizing and destabilizing economic systems. A brief version of this working paper, focused on the underlying theory, appeared as an invited feature article in the February 2020 Society for Chaos Theory in Psychology and the Life Sciences newsletter (Hastings et al. 2020).

KEYWORDS: Stability; Complexity; May-Wigner; Noise; Subprime Crisis; Liquidity Shock

JEL CLASSIFICATIONS: C02; C62; E17; H12

1. INTRODUCTION

The subprime crisis of 2007 generated a near worldwide “great recession,” spread by the world financial network. May, Levin, and Sugihara’s (2008) widely cited paper “Ecology for bankers”[1] (see also May’s keynote[2]) argued that the study of ecosystem dynamics could provide an important framework for analyzing the dynamics and, in particular, the stability of economic networks.

The study of ecosystem dynamics had provided conflicting answers to the question: “Will a large complex system be stable?” (the title of Robert May’s widely cited 1972 paper). Robert MacArthur (1955) and G. Evelyn Hutchinson (1959) had previously argued that stability increases with increasing complexity, whereas May argued that stability decreases with increasing complexity.[1]

As economic systems become more complex, it becomes more relevant to translate stability results and more generally network dynamics from ecosystems to economic systems.

1.1. The Econo-system: Economics = Ecology + Currency

As a working *ansatz*, we quote from May, Levin, and Sugihara (2008):

> There is common ground in analysing financial systems and ecosystems, especially in the need to identify conditions that dispose a system to be knocked from seeming stability into another, less happy state […] “Tipping points,” “thresholds and breakpoints,” “regime shifts”—all are terms that describe the flip of a complex dynamical system from one state to another. For banking and other financial institutions, the Wall Street Crash of 1929 and the Great Depression epitomize such an event. These days, the increasingly complicated and globally interlinked financial markets are no less immune to such system-wide (systemic) threats. *Who knows, for instance, how the present concern over sub-prime loans will pan out?* (emphasis added; we all learned a year later)

Remarks. A note on style. We will make significant use of direct quotations in order to accurately capture historical developments as well as the thought of key leaders and opinion makers.

[1] See, e.g., the historical discussion in Bersier (2007).

In this paper we shall develop and analyze the relationship between the complexity of economic networks and their stability, beginning with a discussion and resolution of the apparent conflict between May (1972) on the one hand and MacArthur (1955) and Hutchinson (1959) on the other. Two disclaimers first:

1. We do not claim to provide an answer to the important question of how to design stable economic systems, only a framework for that discussion.
2. We do not aim to provide a complete history of the stability-complexity discussion, only our personal selection of important highlights. Instead we refer the reader to Allesina and Tang (2015) for a good recent history.

We again quote May, Levin, and Sugihara (2008):

> Catastrophic changes in the overall state of a system can ultimately derive from how it is organized—from feedback mechanisms within it, and from linkages that are latent and often unrecognized. The change may be initiated by some obvious external event, such as a war, but is more usually triggered by a seemingly minor happenstance or even an unsubstantial rumour. […] For instance, to what extent can mechanisms that enhance stability against inevitable minor fluctuations, in inflation, interest rates or share price for example, in other contexts perversely predispose towards full-scale collapse?

1.2. The Hutchinson and MacArthur–May Debate: The Role of Scaling

Our discussion (Hastings 1983) (see section 2.3) of the conflict between Hutchinson (1959) and MacArthur (1955) (complexity → stability) and May (1972) (complexity → instability) focuses on how the strength of interactions in a system scales with the number of interactions in a system (complexity of the underlying graph). Whereas May (1972) considered the stability of arbitrary complex systems, Hutchinson (1959) and MacArthur (1955) argued that complexity in the form of multiple pathways for a fixed energy flow enhances stability; roughly, each pathway is less important and disruption of one pathway leaves others intact. In order to more carefully compare these viewpoints, we introduce the following definitions and notation.

1.2.1 Definitions and Notation

System. For now, we shall consider *linear, discrete-time* systems of the form

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) \tag{1}$$

where $\boldsymbol{x}$ is a column vector with n rows and A is a $n \times n$ matrix, typically not symmetrical. We shall refer to n as the size of the system.

Underlying graph. The *underlying graph* of system (1) consists of n nodes, one for each component of the system, together with a directed edge from node j to node i if component j affects component i, that is, if $A_{ij} \neq 0$.

Degree of a vertex. The *in-degree* of vertex i in the underlying graph of linear, discrete-time system (1) is the number of nonzero entries in the i^{th} row of the matrix A, that is, the number of components which affect the i^{th} component of the system.

Connectance. The *connectance* C of the matrix A is defined by the equation $C = k/n$, where k is the average in-degree. Essentially, the connectance is the fraction of entries in the matrix A that are nonzero.

Mean interaction strength. The mean interaction strength α is the size, suitably defined, of interactions in the system, that is, nonzero entries in the matrix A.

May (1972) used the following specific form of large random complex systems to relate stability of a system to its size, connectance, and mean interaction strength. Let X be a random variable with a symmetric distribution of expectation 0 and second moment 1.[2] Then the entries of A are chosen independently identically distributed to be αX with probability C, and 0 otherwise.[3]

[2] The proof requires additional bounds on the distribution X, such as that its fourth moment is finite. See section 2.2, below.

[3] Such random graphs in which edges are added independently are known as Erdős-Rényi random graphs. More general random graphs and matrices are discussed in section 2.2.

Then asymptotically almost surely as $n \to \infty$, the spectral radius (size of dominant eigenvalue) of A approaches $\alpha\sqrt{nC}$. Thus, for large n, the system $\boldsymbol{x}(t+1) = A\boldsymbol{x}(t)$ is stable provided $\alpha\sqrt{nC}$ < 1, and unstable otherwise. This result, the May-Wigner stability theorem, provides a formal, mathematical setting for the concepts "too big," and "too connected."

Too big, too connected. For fixed mean interaction strength α, the size of the largest eigenvalue increases with increasing size or connectance, and thus stability decreases with increasing complexity (May 1972). The system ultimately becomes too big or too connected for stability, namely:

$$nC > 1/\alpha^2 \tag{2}$$

Multiple pathways, diversification. MacArthur (1955) and Hutchinson's (1959) model considered energy flows through a complex network, with the energy flow into a node divided among the pathways into that node. If we consider the edges incident on a given node, the mean energy per edge scales inversely with the in-degree (number of edges incident on) k of that node. Since

$$k = nC \tag{3}$$

the mean energy flow into a typical node scales inversely with the product of the size and connectance of the system. MacArthur (1955) and Hutchinson (1959) assume that increasing size or connectance provides multiple (independent) pathways for flows through the network, resulting in a decrease in average interaction strength α. If α were to decrease sufficiently quickly with increasing size or connectance, so that

$$\alpha^2 nC < \text{const} < 1 \text{ for sufficiently large } k = nC \tag{4}$$

then stability would increase with increasing complexity.

As we shall see, although increasingly linked and complex structures may be assembled/ may evolve so as to enhance (Lyapunov) stability, these very linkages have the potential (realized in the subprime loan crisis) to cause collapse (reduced structural stability[4]) (Hastings 1984). One might expect that diversification in finance (for example, in stock portfolios, mortgage pools, linkages among banks) would work the same way, at least as long as (financial) flows remain bounded and interactions remain uncorrelated (as in May's "random matrices"). In fact, analogous diversification is the basis of Markowitz's (1952) "modern portfolio theory." Markowitz defines an efficient strategy to be one which maximizes expected returns for a given variance of return (a measure of "risk"?), or equivalently, which minimizes variance for a given expected return.

This "Gaussian" philosophy leads to "value at risk" models (Jorion 1997), "sanctioned for determining market risk capital requirements for large banks by US and international banking authorities through the 1996 Market Risk Amendment to the Basle Accord," However, "'reduced-form' or 'time-series' models of portfolio P&L [profit and loss] cannot account for positions' sensitivities to current risk factor shocks or changes in current positions. However, their parsimony and flexibility are convenient and accurate for modeling the mean and variance dynamics of P&L" (Berkowitz and O'Brien 2002).

But what if major risks are unanticipated (and thus not part of the design/evolution of the system), and not adequately characterized by Gaussian distributions?

1.3. From the subprime loan crisis to the Great Recession: An Emergent Problem

Many leading economists failed to predict that the subprime loan crisis would spread into a virtually worldwide Great Recession (May, Levin, and Sugihara 2008; Krugman 2009; Rajan 2010). In 2007, it appeared that the "state of macro is good" (Blanchard 2009) (the projected scale of the subprime loan crisis is discussed later in this section).

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[4] Roughly, a system is structurally stable if small changes in parameters cause only small changes in dynamics; see section 2.4, below.

However, as we shall see below, it can be hard to determine whether a system is stable, or more generally to understand the tails (Taleb 2010) of its dynamics (Caballero 2010). But rare events can be extremely important, even catastrophic—such as Taleb's (2007) "black swan":

> The Black Swan is defined here as a random event satisfying the following three properties: large impact, incomputable probabilities, and surprise effect. First, it carries upon its occurrence a disproportionately large impact. The impact being extremely large, no matter how low the associated probability, the expected effect (the impact times its probability), if quantified, would be significant. Second, its incidence has a small but incomputable probability based on information available prior to its incidence [Arrow 1987]. Third, a vicious property of a Black Swan is its surprise effect: at a given time of observation there is no convincing element pointing to an increased likelihood of the event.

In particular, Taleb offers the following blast at modern portfolio theory:

> But the [Nobel Memorial Prize in Economic Sciences] committee has gotten into the habit of handing out Nobel Prizes to those who "bring rigor" to the process with pseudo-science and phony mathematics. Harry Markowitz and William Sharpe, who built beautiful Platonic models on a Gaussian base, contributing to what is called Modern Portfolio Theory. Simply, if you remove their Gaussian assumptions and treat prices as scalable, you are left with hot air.

1.4. What Happened?

Clearly the subprime crisis was not contained. Instead one saw a systemwide breakdown (Haldane and May 2011; Markose et al. 2012; Hellwig 2009; Mat and Arinaminpathy 2009; Battison et al. 2016). Dymski (2009) describes the breakdown in terms of Minskyan (1975, 1986, 1993) dynamics:

> financing decisions in the economy, financial-market dynamics and macroeconomic growth are interlinked. Minsky argues that economic units move systematically from "robust" financial positions, with minimal credit outstanding and little leverage, toward "fragile" and then "Ponzi" financial positions, which leave them increasingly unable to meet debt obligations taken on due to overoptimistic expectations. Eventually, cash-flow constraints bind and slow the expansion. Then expectations break down and asset values fall; if unchecked, a debt deflation process may be unleashed.

One might even consider the failure to be an emergent phenomenon—one not predictable from local dynamics, but rather one which "emerges" from interactions in an unpredictable way (Arrow 1987).

It appears that the financial network, "designed" in some sense to be stable, became too big and too complex (Haldane and May 2011; Battiston et al.; Haldane 2013) and, in particular, too connected (Markise et al. 2012; León et al. 2011), and thus served as a substrate to transmit the shock caused by the failure of Lehman (Allington, McCombie, and Pike 2012), which then resulted in a financial freeze-up and near collapse, a network realization of Minsky's dynamics.[5] In Allington, McCombie, and Pike (2012) they argue that: "the major problem was the assumption that the future could be modelled in terms of Knightian risk (as in the rational expectations and efficient markets hypotheses). It is shown that the near collapse of the banking system in the advanced countries was due to a rapid increase in radical uncertainty."

It is interesting and potentially important to recall the definition of *ergodicity* here. One can regard the time series of key economic state variables tracing out a trajectory in an economic state space, the state space of the economy as a dynamical system. This dynamical system is ergodic if "most" trajectories pass near "most" points in the economic state space, defining an underlying probability measure for computing the probability of a measurable subspace of the economic state space. Thus, for ergodic systems, the probability of seeing a given event can be approximated by the probability that it has occurred in the past, along a trajectory in economic state space—a white swan, not Taleb's black swan. Ergodicity may fail for many reasons, but we cite here two important reasons:

(1) The past history (trajectory) is too short—there is little evidence of the 500-year storm in a 50-year history.
(2) The underlying dynamics themselves may have changed.

For the latter, Allington, McCombie, and Pike argue for increases in the sensitivity of the financial system due to "amplifying effects of the large increase in leverage of the banks that had occurred over the last two decades" (see, for example, Blanchard [2009], Brunnermeier [2009], Rajan [2005, 2010], and Roubini and Mihm [2010]).

[5] See also Raddatz (2010), Dooley and Hutchison (2009), and Hesse, Frank, and González-Hermosillo (2008).

At this point, the failure of Lehman provided *an unanticipated shock* to the system: "Bankruptcy counsel for Lehman Brothers, Harvey Miller, argued that hedge funds 'expected the Fed to save Lehman's based on the Fed's involvement in LTCM's rescue. That's what history has taught them.'" (FCIR 2011, 58).[6]

1.5. A Key Observation

As described above, complexity can make a system more resilient (as measured by Lyapunov stability), but also susceptible to failure under an unanticipated shock. As Bardoscia et al. (2017) stated:

> Here we show how processes that are widely believed to stabilize the financial system, that is, market integration and diversification, can actually drive it towards instability, as they contribute to create cyclical structures which tend to amplify financial distress, thereby undermining systemic stability and making large crises more likely. This result holds irrespective of the details of how institutions interact, showing that policy-relevant analysis of the factors affecting financial stability can be carried out while abstracting away from such details… The application of network theory to finance (Schweitzer 2009) has made it clear that complexity can destabilize the financial system (Stiglitz 2010, Brock 2009, Battiston et al. 2012, Arinaminpathy et al. 2012).

In particular, Stiglitz (2010) makes the point that "well-designed networks have circuit breakers, to prevent the 'contagion' of the failure of one part of the system from spreading to others."[7] Circuit breakers (Schwert 1998; Harris 1998; Subrahmanyam 1994) have been used to (attempt to) arrest precipitous market declines. Since, as Arinaminpathy, Kapadia, and May (2012) observed, "more hedging instruments may destabilize markets," is it possible for one to design workable network-level circuit breakers?

1.6. Too Big to Fail? Too Interconnected to Fail?

In a 30,000-foot view, the financial network was allowed to become too complex (too connected, see section 2.3) and increasing leverage reduced fault tolerance (the "distance to a tipping point," see section 2.5), at which point the Lehman failure triggered a cascade of freeze-ups.

[6] See Allington McCombie, and Pike (2012, fn 10).
[7] Compare the modular structure of power grid, as in section 2.6, below; see also McCall (2020).

Hellwig (2009) provides a deeper dive into how the Lehman failure triggered the Great Recession, and in particular the failure of mortgage securitization to properly allocate risk:

> Excessive maturity transformation through conduits and structured-investment vehicles (SIVs); when this broke down in August 2007, the overhang of asset-backed securities that had been held by these vehicles put significant additional downward pressure on securities prices. Second, as the financial system adjusted to the recognition of delinquencies and defaults in US mortgages and to the breakdown of maturity transformation of conduits and SIVs, the interplay of market malfunctioning or even breakdown, fair value accounting and the insufficiency of equity capital at financial institutions, and, finally, systemic effects of prudential regulation created a detrimental downward spiral in the overall financial system. The paper argues that these developments have not only been caused by identifiably faulty decisions, but also by flaws in financial system architecture. In thinking about regulatory reform, one must therefore go beyond considerations of individual incentives and supervision and pay attention to issues of systemic interdependence and transparency.

An illustrative quotation from *Liar's Poker* (Lewis 2010), a historical novel about the dynamics leading up to Black Monday, provides an interesting summary of what might go (and had gone) wrong:

> We had overlooked the need to obtain the approval of the German government [to create warrants on German interest rates, and the opportunist saved us from embarrassment. The German government has no say in the Euromarkets. The beauty of the Euromarket was that it fell under no government's jurisdiction. We could, in theory, have ignored the Germans. But we had to be polite. Salomon Brothers hoped to open an office in Frankfurt, and the last thing the firm needed was angry German politicians. […] So the opportunist became our emissary to the German Finance Ministry. He persuaded the authorities that our deal would neither undermine their ability to control their money supply (true) nor encourage speculation in German interest rates (*false; the whole point was to encourage speculation*.) (emphasis added)

Mac Arthur (1955) and Hutchinson (1959) versus May (1972), reprise, translated loosely to economics. The Mac-Arthur-Hutchinson argument that the existence of multiple pathways for energy flow enhances stability is similar to conventional arguments for portfolio diversification in economics. For example, one expects a generalist predator with many (substitutable) food sources (prey) to be less affected by the failure of one food source than a specialist predator dependent upon one or a few food sources; similarly, one expects a more diverse investment portfolio to display less volatility than a less diverse portfolio.

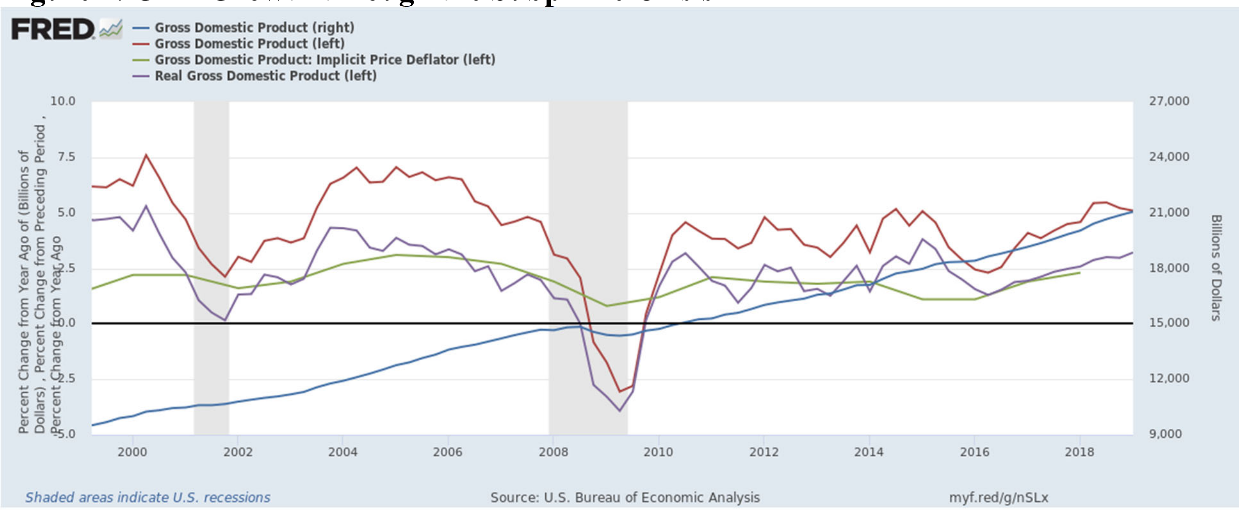
The effectiveness of diversity in reducing volatility of course depends upon diversity increasing the effective number of degrees of freedom in the system. The increase in effective number of degrees of freedom can be illusory or even false in the presence of systemic or correlated shocks; for example, a national decline in housing prices affects the solvency of whole portfolios of mortgages (the subprime crisis) and liquidity shocks such as those caused by the collapse of Lehman can destabilize the whole financial network (see Dooly and Hutchinson [2009], Hesse Frank, and González-Hermosillo [2008], and Raddatz [2010]).

Finally, large shocks can cause tipping points where the whole dynamics change (May, Levin, and Sugihara (2008). In section 2, we shall relate the potential for tipping points to extensions of May's classic result on the stability of large systems (May 1972): the May-Wigner stability theorem.

Projected scale of the subprime loan crisis. Projected losses were similar to losses from the savings and loan (S&L) crisis, which did not generate a worldwide recession. In May 2007, the subprime crisis had been projected to cause losses of $300 billion, an estimate which later rose to $350–420 billion (Blundell-Wignall 2008). In comparison, losses from the S&L crisis were estimated between $152.9 billion (Curry and Lynn 2000) and $160 billion (Gramling 1996). In both cases, projected losses represent 2–3 percent of GDP.

One way to understand the size of these projected losses is to compare them to the US GDP, which was $14.383 billion in 2007Q2 and $14.681 billion on the eve of the subprime crisis in 2007Q3 (BEA 2019a). Projected losses thus represented 2–3 percent of US GDP, at a time when nominal GDP was growing at ~4 percent per year and real GDP was growing at ~2 percent per year (figure 1). Losses in the S&L crisis represented 2.7 percent of US GDP of $5.873 billion in 1990Q1 (roughly the midpoint of the resolution of the S&L crisis) and 2.1 percent of 1995Q1 GDP of $7.522. Thus, projected losses in the subprime crisis were comparable in relative size to losses in the S&L crisis.

Figure 1. GDP Growth through the Subprime Crisis



Notes: GDP, seasonally adjusted annual rate (BEA 2019b). GDP: Implicit Price Deflator (A191RI1A225NBEA), annual data, percent change from previous year (BEA 2019c).
Real GDP (GDPC1): billions of chained 2012 dollars, seasonally adjusted change over previous year (BEA 2019d).
For more information see the "Guide to the National Income and Product Accounts of the United States (NIPA)."

The rest of this paper is organized as follows. Section 2 discusses the stability-complexity debate. We explore the relationship between stability and complexity further, formalize the May-Wigner stability theorem, discuss the probability of collapse when noise is added, and finally consider extensions to nonlinear dynamics. Section 3 discusses the implications for economics, and section 4 ("It's a Small World") presents random matrix models for the world trade network. We show that the world trade network is "effectively small" and discuss how this limits the ability to forecast stability properties. We conclude in section 5 with a discussion of our results.

2. THE STABILITY-COMPLEXITY DEBATE

This relatively long section is organized as follows. We first compare MacArthur (1955) and Hutchinson's (1959) argument that complexity increases stability (section 2.1) with May's (1972) argument (the May-Wigner stability theorem) that complexity decreases stability (section 2.2). We then seek to resolve this conflict by invoking a following scaling argument based on MacArthur (1955) and Hutchinson (1959). The average interaction strength of a component of a network (food web or economic network) that interacts with many components should do so more weakly than that of a component with fewer interactions (section 2.3). We then review various definitions of stability (section 2.4). This discussion is followed by an analysis of the effect of adding noise to the linear systems considered by May (section 2.5), with applications to survival times (persistence) of these systems, and a concluding summary (section 2.6).

2.1. Complexity Increases Stability: MacArthur and Hutchinson

First, a 30,000-foot view of stability and complexity of ecosystems finds a general trend (see May (2001) and references therein): increased complexity correlates with reductions in variability (table 1). One may take variability as a measure of instability; in which case lower variability corresponds to increased stability.[8]

Table 1. Stability and Complexity of Ecosystems

System →	Tropical	Mid-latitude	Northern
Complexity	High	Intermediate	Low
Variability	Low	Intermediate	High

MacArthur and Hutchinson offered an explanation. We quote from Hutchinson's 1958 Presidential Address to the American Society of Naturalists (Hutchinson 1959):

[8] See Harrison (1979) and Pimm (1984), who discuss a variety of stability concepts; see also section 2.4.

> Recently MacArthur (1955) using an ingenious but simple application of information theory has generalized the points of view of earlier workers by providing a formal proof of the increase in stability of a community as the number of links in its food web increases. MacArthur concludes that in the evolution of a natural community two partly antagonistic processes are occurring. More efficient species will replace less efficient species, but more stable communities will outlast less stable communities. In the process of community formation, the entry of a new species may involve one of three possibilities. It may completely displace an old species. This of itself does not necessarily change the stability, though it may do so if the new species inherently has a more stable population (cf. Slobodkin, 1955) than the old. Secondly, it may occupy an unfilled niche, which may, by providing new partially independent links, increasing stability. Thirdly, it may partition a niche with a pre-existing species.

In essence, MacArthur (1959) argued that an increase in complexity of a food web provides additional links for energy flow, and fluctuations in flow along any link have less effect upon overall dynamics.

2.2. "Will a Large Complex System Be Stable?" (May 1972)

Following seminal work by Wigner (1953, 1958) and observations of Gardner and Ashby (1970), May (1972) asked: "Will a large complex system be stable?" and interpreted this question as asking about the Lyapunov stability of a system of first order differential equations

$$\frac{dx}{dt} = \boldsymbol{f}(\boldsymbol{x}) \qquad (5)$$

with an equilibrium at $\boldsymbol{x} = \boldsymbol{x}_0$. If $\boldsymbol{f}(\boldsymbol{x})$ is sufficiently smooth, we can linearize system (5) about the equilibrium, and then replace $\boldsymbol{x} - \boldsymbol{x}_0$ with $\boldsymbol{x}$, obtaining the linear system

$$\frac{dx}{dt} = F\boldsymbol{x} \qquad (6)$$

Here F denotes the Jacobian matrix of $\boldsymbol{f}$ evaluated at $\boldsymbol{x} = \boldsymbol{x}_0$.

The dynamics of the system (5) are approximated by the dynamics of its linear approximation (6) near equilibrium. Moreover, both systems have equivalent stability properties near their respective equilibria $\boldsymbol{x} = \boldsymbol{x}_0$ and $\boldsymbol{x} = \boldsymbol{0}$, respectively. These equilibria are stable if and only if all eigenvalues of F have negative real part.

The system (5) yields two different discrete-time systems. The first,

$$\boldsymbol{x}(t + \Delta t) = exp(\Delta t\ F)\ \boldsymbol{x}(t) \quad (7)$$

is obtained by solving system (2) analytically and then evaluating the solution at times spaced at intervals of Δt.

The second,

$$\boldsymbol{x}(t + \Delta t) = (I +\ \Delta t\ F)\ \boldsymbol{x}(t) \quad (8)$$

is obtained by solving system (2) with the Euler method, or equivalently expanding $exp(\Delta t\ F)$ as a Taylor series in Δt, and deleting higher order terms in Δt.

System (7) is Lyapunov stable if and only if all eigenvalues of the matrix

$$A = exp(\Delta t\ F) \quad (9)$$

lie strictly within the unit circle, that is,

$$\parallel \lambda \parallel < 1 \quad (10)$$

and similarly, system (8) is Lyapunov stable if and only if all eigenvalues of the matrix

$$A' = \ (I +\ \Delta t\ F) \quad (11)$$

lie strictly within the unit circle. These criteria are essentially the same for small "time steps" Δt, but different for large time steps. Criterion (9) requires only that the real part of all eigenvalues of the matrix A be negative, whereas criterion (11) will fail for real parts significantly below 0 ($Re(\lambda) < -2/\Delta t$ will yield exponentially increasing oscillations).[9]

May then considers random matrices *A* whose entries are chosen independently as follows:

$$A_{ij} = \begin{cases} 0 & \text{with probability } 1 - C \\ \alpha X & \text{with probability } C \end{cases} \tag{12}$$

where *X* is a random variable with expectation 0 and second moment 1. Note that the underlying graph of *A*, that is a graph with *n* nodes and an edge from node *j* to node *i* whenever $A_{ij} \neq 0$, is an Erdős-Rényi random graph of connectance *C*. Then May argued that for large *n*, the spectral radius of *A* approaches $\alpha\sqrt{nC}$ implying the asymptotic stability criterion of equation (2) above:

$$\alpha^2 nC < 1 \tag{13}$$

2.2.1 Proofs and Counterexamples

However, as Cohen and Newman (1984, 1985) showed, the May-Wigner stability criterion (2) is false without additional hypotheses on bounds for the higher moments of the distribution *X*. Several authors (Girko 1985; Bai and Yin 1986; Geman 1986) proved the May-Wigner criterion under these hypotheses, for example, Bai and Yin (1986), who require only a bound on the fourth moment of *X*. Under these hypotheses, May's stability criterion (9) only holds asymptotically almost surely (a.a.s), that is, with probability approaching one as the size of the system approaches ∞, for fixed connectance and α.

2.2.2 More General Networks

The graphs (or networks) underlying the May-Wigner stability theorem are known as Erdős-Rényi random graphs, edges (nonzero entries in the matrix) are chosen independently with equal probability (Erdös and Rényi 1961). See Erdös and Rényi (1959), Gilbert (1959), and Bollobás (2001) for a general discussion of random graphs.

[9] Consider, for example, the one-dimensional system $x(t + \Delta t) = (1 + r\Delta t)\, x(t)$. Stability requires that $|1 + r\Delta t| < 1$, and thus both the conditions: $r < 0$ and a second condition requiring that the time delay not be too large compared with the time scale $1/|r|$, namely that $|r\Delta t| < 2$. Please see further discussion below.

2.3. Complexity May or May Not Increase Stability

"Early studies suggested that simple ecosystems were less stable than complex ones, but later studies came to the opposite conclusion. Confusion arose because of the many different meanings of 'complexity' and 'stability.' Most of the possible questions about the relationship between stability-complexity have not been asked. Those that have yield a variety of answers" (Pimm 1984), in some cases depending on the particular definition of stability (Harrison 1979, Pimm 1984); see section 2.4, below.

In MacArthur's (1955) "multiple energy pathways" scenario, the flow into a node is distributed over the edges into that node (in-degree), and therefore the average flow along each edge scales inversely with the in-degree. More generally, in ecology, a species interacting with many species can be expected to interact more weakly with each one than a species interacting with fewer species, thus α decreases with increasing $k = nC$. If, as MacArthur's information theoretic analysis suggested,

$$\alpha = {}^{const}/_{nC} \tag{14}$$

then

$$\parallel \lambda \parallel = \alpha\sqrt{nC} = {}^{const}/_{\sqrt{nC}} \rightarrow 0 \text{ as } k = nC \rightarrow \infty \tag{15}$$

and Lyapunov stability increases with increasing complexity. More generally, if α scales inversely with some power of nC, for example,

$$\alpha = {}^{const}/_{(nC)^s} \tag{16}$$

then stability increases with increasing complexity if $s > 1/2$, and stability decreases with increasing complexity if $s < 1/2$; see Hastings (1984) and also Pimm (1984) for constant $k = nC$.

2.4. Stability Concepts, Structural Stability versus Complexity

We shall briefly review stability concepts before returning to the stability-complexity debate from the previous section.

2.4.1 Stability Concepts

Harrison (1979) compared four concepts of "stability under environmental stress: resistance, resilience, persistence, and variability."[10] To this list, we add two more mathematically oriented stability concepts: Lyapunov stability (c.f. Holmes and Shea-Brown 2006) and structural stability (c.f. Pugh and Peixoto 2008).

Table 2. Stability Concepts

Concept	**Definition**	**Remarks**
Resistance (to environmental stress)	A more resistant system has a smaller response	More resistant systems tend to be more persistent. "Friction" may increase resistance but also decreases resilience.
Resilience (in the presence of environmental stress)	A more resistant system (with a steady state) returns more quickly to the steady state	More resilient systems tend to be more persistent. Increasing Lyapunov stability increases resilience.
Persistence	A more persistent system is less likely to fail in the presence of environmental stress.	Resistance and resilience both favor persistence. Another important factor is how much variability the system can handle before failing. It can be difficult to assess persistence (Connel and Sousa 1983).
Variability	One may regard less variable systems as more stable.	Resistant systems tend to have less variability, but low variability may also reflect lower environmental stress (smaller and/or fewer shocks). More resilient systems display less variability in the presence of a series of shocks.
Lyapunov stability (near a steady state)	Lyapunov stability measures the rate of return to steady state with eigenvalues of the corresponding Jacobian matrix.	The May-Wigner stability theorem assesses estimated Lyapunov stability. Increased Lyapunov stability is associated with increased resilience, and less variability (Hastings 1984; Schriber and Hastings 1995).
Structural stability	Structural stability measures the effect of changes in model parameters, or even in the model itself; in contrast to Lyapunov stability which measures the effect of perturbing state variables.	One major perturbation involves correlation among interaction terms; see Junior and Franca (201) for the effect of correlation in financial markets. More generally, liquidity shocks can destabilize the financial network; see Dooley and Hutchinson (2009), Hesse, Frank, and González-Hermosillo (2008), and Raddatz (2010).

[10] See also Pimm (1984) and Holling (1973).

2.4.2 Scaling and the Stability-Complexity Debate, Revisited.

Our discussion of scaling and stability above assumed random interactions with a 0 mean. What happens if the system is perturbed so as to make the mean interaction nonzero? This is a question considered briefly above and more thoroughly in Stone's (2016) paper "When Google meets Lotka-Volterra."

We begin with a simulation. For the mean 0 case of the May-Wigner stability theorem, let A be a16×16 random matrix whose entries A_{ij} are chosen independently from a uniform distribution on the interval $(-0.5, 0.5)$. For the case of nonzero mean μ, the entries A_{ij} are chosen independently from $U(\mu - 0.5, \mu + 0.5)$, a uniform distribution on the interval $(\mu - 0.5, \mu + 0.5)$. The resulting matrices are among the simplest Google matrices (Brin and Page 1998; Ding and Zhou 2007) because of their role in Google's page rank computations. This changes the expected row sum or column sum to 16μ, in which case the Gershgorin circle theorem (c.f. Weisstein 2003) predicts that the expected row sum is an approximate bound to the size of the dominant eigenvalue. We found the spectral radius with the power law in Python and ran 10,000 replicates for each set of parameters. Results are shown in table 3, in each case showing the mean and standard deviation of the size of the dominant eigenvalue.

Table 3. Effect of Perturbing Random Interactions Making the Mean Nonzero

Mean μ	Spectral radius
0	1.2402 ± 0.1611
0.031	1.2524 ± 0.1669
0.062	1.3336 ± 0.2001
0.094	1.5729 ± 0.2888
0.125	1.9873 ± 0.3494
0.188	2.9888 ± 0.3207
0.25	4.0000 ± 0.3033

Notes: Small perturbations (expected row sum = 0.5, $\mu = 0.5/16 = 0.03125$), that is, $A_{ij} = U(-0.46875, 0.53125)$, have almost no effect on the spectral radius, as in Stone's (2016) asymptotic results. The spectral radius increases for intermediate perturbations ($0.062 \leq \mu \leq 0.094$) and finally is very close to the expected row sum for larger perturbations ($\mu \geq 0.125$, expected row sum $= 16\mu \geq 2$), as expected from the Gerschgorin circle theorem.

The above change in eigenvalues may reflect a change in stability after all entries are scaled so as to make the spectral radius < 1 in the case of 0 mean (May-Wigner), but > 1 after perturbation, a form of structural instability. More generally, for asymptotically large systems, Lyapunov stability increases with complexity (here size, connectance, or both), provided that the mean interaction strength α satisfies the scaling rule:

$$\alpha = o\left(1/\sqrt{nC}\right), \text{ that is, } \lim_{k \to \infty} \alpha\sqrt{nC} = 0 \tag{17}$$

In contrast, structural stability requires a more rapid decrease in α with increasing complexity, namely

$$\alpha = o(1/(nC)), \text{ that is, } \lim_{k \to \infty} \alpha(nC) = 0 \tag{18}$$

For intermediate values of scaling, e.g.,

$$\alpha = o(1/(nC)^s) \text{ , with } 1/2 < s < 1 \tag{19}$$

increasing complexity increases Lyapunov stability, but decreases structural stability. More generally, a system may approach the limits of stability in one environment, but become fragile with respect to "exogenous" forces that change the environment and thus the dynamics.

2.5. Stochastics

Here we review and update the results of Hastings (1984). What happens if we add "noise" to the basic model

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t),$$

obtaining the system of stochastic difference equations

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) + \sigma\Delta\boldsymbol{w}(t) \tag{20}$$

Here the increments $\Delta \boldsymbol{w}(t)$ are independent and identically distributed (i.i.d.) Gaussian distributions with mean 0 and variance 1 (the increments in a discrete time random walk = discrete time Brownian motion).

For large t, equation (20) implies

$$\boldsymbol{x}(t) = \sigma(\Delta \boldsymbol{w}(t-1) + A\,\Delta \boldsymbol{w}(t-2) + A^2 \Delta \boldsymbol{w}(t-3) + \cdots + A^{t-1} \Delta \boldsymbol{w}(0)) \qquad (21)$$

Equation (20) is an AR(1) or discrete-time Ornstein-Uhlenbeck (Uhlenbeck and Ornstein 1930) process.

2.5.1 *Falling off a Financial Cliff*

Suppose an economic system "falls off a cliff" if $\| \boldsymbol{x}(t) \|$ becomes too large. Following Hastings (1974), we compute the long-term behavior of $\| \boldsymbol{x}(t) \|$. Since the increments $\Delta \boldsymbol{w}(t)$ are i.i.d. $\mathrm{N}(0,1)$, replacing A^k by $\| \lambda \|^k \approx \left(\alpha\sqrt{nC}\right)^k$ implies that the long-term distribution of $\boldsymbol{x}(t)$ is given by a normal distribution of mean 0 and variance ${\sigma^2}/{1-\alpha^2 nC}$ provided $\alpha^2 nC < 1$ (the variance approaches ∞ as $\alpha^2 nC \rightarrow 1$). For $\alpha^2 nC < 1$, the distance to the cliff, denoted s, normalized by the long-term standard deviation of $\| \boldsymbol{x}(t) \|$, namely

$$s\sqrt{1-\alpha^2 nC}/\sigma \qquad (22)$$

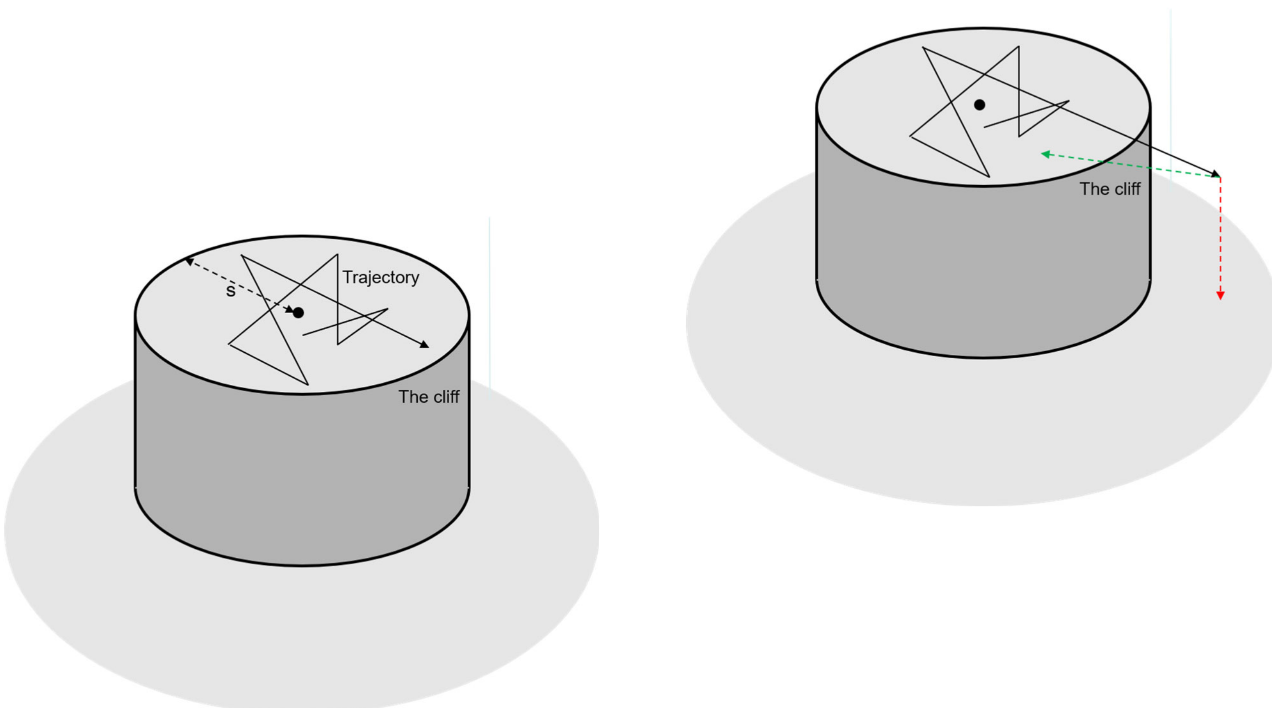
provides a measure of the likelihood of collapse: collapse is highly unlikely if

$$s\sqrt{1-\alpha^2 nC}/\sigma \geq 5 \qquad (23)$$

a "5σ" deviation.[11]

[11] 5σ or more is appropriate given the importance of avoiding collapse and the challenge of estimating parameters and tails of distributions. For example, if the trajectory in the economic state space has autocorrelation = 0 after 2 years (c.f. Hastings, Young-Taft, and Wang 2019), the probability of a 3σ event in 10 years is almost 1 percent.

Figure 2. Falling off a Financial Cliff



Notes: *Left:* A cartoon view of a trajectory governed by an Ornstein-Uhlenbeck process with an "absorbing barrier" at $\| \boldsymbol{x}(t) \| = s$, shown here as a cliff. *Right:* The "Wile-e-coyote" effect: a temporarily insolvent firm may survive and become solvent (green arrow) or fail (red arrow), depending upon creditor and investor confidence, analogously for governments (the effects of large debt as a percentage of GDP depend upon confidence in the ability to avoid default, see IMF data as reported in World Population Review [2020]).[12]

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[12] Predictions of a "fiscal cliff" can be difficult and are often wrong. Consider the following 2012 prediction from the Fiscal Times:

> The Bank of America [B of A] economists note that the prospects of lawmakers forging a broad deal during the lame duck session are dim—and that the chatter that we could go over the cliff, if only briefly, without serious consequences could be dangerous for investors and the economy.
>
> The "consumer confidence could dry up quickly" if companies decide to postpone hiring as the attention turns to the fiscal cliff and the full scope of the quagmire in Washington becomes more apparent. "We continue to see only limited scope for some sort of action during the lame duck session of Congress that would avoid most of the cliff—especially if the election largely returns the status quo that created it," B of A economist Michael Hanson wrote.

A deeper analysis shows that the first passage time to a "cliff" at $\boldsymbol{x}(t) = s$ for the analogous continuous-time Ornstein-Uhlenbeck process

$$dx = -rxdt + \sigma dw \tag{24}$$

where dw denotes the infinitesimal increment in a Wiener process (continuous-time random walk, c.f. Weisstein [2003]); thus $dw = N(0,1)$, a normal distribution of mean 0 and variance 1 scales as follows (Marlin 1975):

$$T \sim (1/r) \exp\left(0.46\, (2rs^2/\sigma^2)\right) \tag{25}$$

2.5.2 Remarks

One can think of σ^2/r as the diffusion rate in natural time units $1/r$; thus σ^2/r has units of length^2. Then rs^2/σ^2 represents the square of the distance to the boundary divided by the normalized diffusion rate.

The parameter r is related to the Lyapunov exponent in the discrete-time process as follows: $\lambda = exp(-r)$, or $r = -ln(\lambda)$, suggesting (caution for small T)

$$T \sim (-1/ln(\lambda)) \exp\left(-0.46\, (2ln(\lambda)s^2/\sigma^2)\right)$$

or more simply

$$T \sim (-1/ln(\lambda))\, \lambda^{-0.92s^2/\sigma^2} \tag{26}$$

Note that the probability of falling off the cliff increases sharply with increasing λ. The eigenvalue λ is related to the natural time scale of return to steady state by the formula

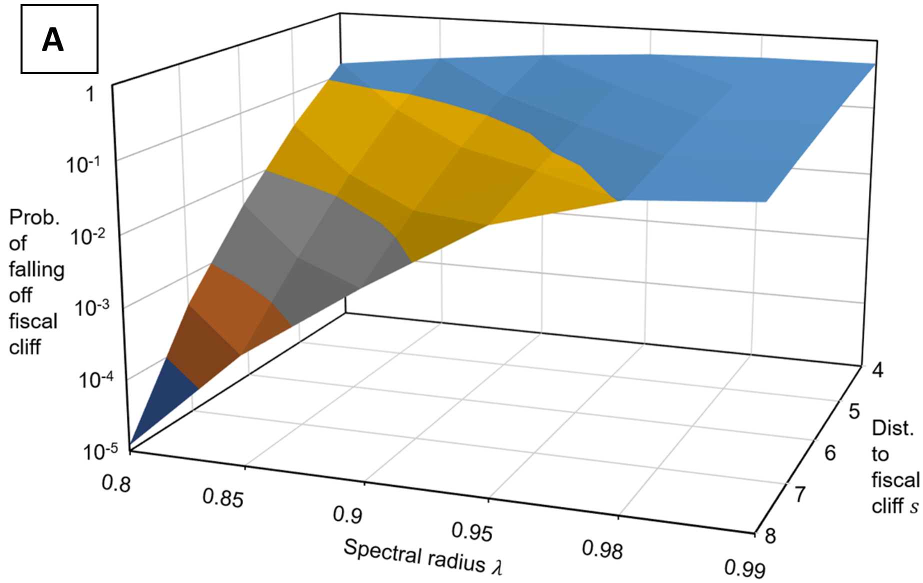
$$\text{time scale} = 1/(1-\lambda) \tag{27}$$

Of course, falling off a (financial) cliff frequently arises as a loss of confidence following a significant financial event. Here is a schematic illustrating the idea of "falling off a (financial) cliff," as well as what we term the "Wile-e-coyote" effect: a bankrupt firm may not fail until confidence is lost (red arrow in figure 2). Alternatively consider how the Lehman bankruptcy triggered the liquidity crisis, which then triggered the Great Recession (Dymski 2009; Allington McCombie, and Pike 2012; Hesse, Frank, and González-Hermosillo 2008; Financial Crisis Inquiry Comission 2011).

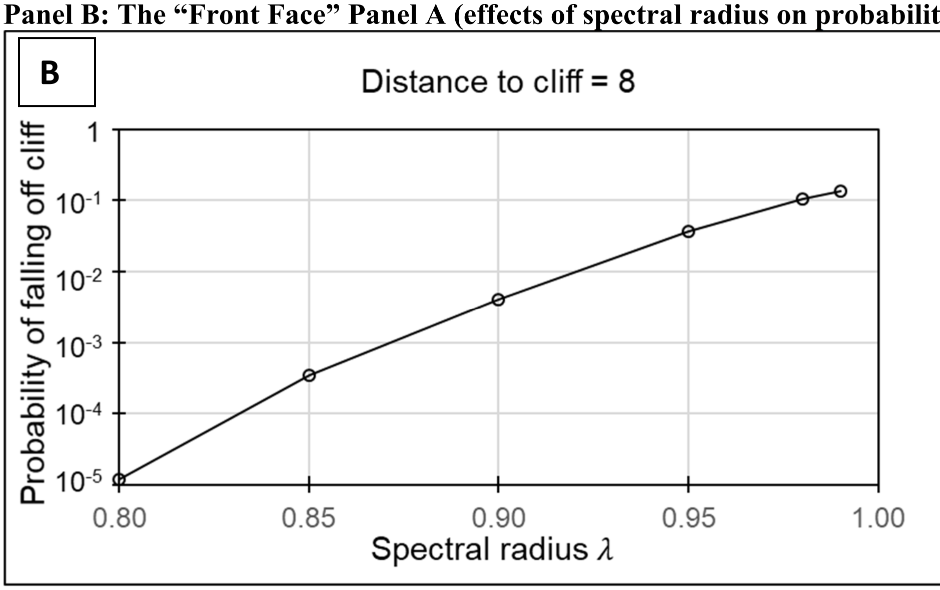
However, large variability makes it "not very useful" in practice (Lindenberg et al. 1975). For that reason, we explore instead how the probability of falling off a cliff scales with key parameters in a one-dimensional Orenstein-Uhlenbeck model (figure 3).

Figure 3. Probability of "Falling off the Fiscal Cliff"

Panel A: Effects of Spectral Radius and Distance to Fiscal Cliff on Probability of Falling Off



Panel B: The "Front Face" Panel A (effects of spectral radius on probability of falling off the fiscal cliff)



Notes: *Methods: we* simulated the one-dimensional AR(1) process

$$x(t+1) = \lambda x(t) + \sigma \Delta w(t), \sigma = 1, x_0 = 0 \tag{17}$$

with an absorbing barrier ("fiscal cliff") at $s = 4, 5, \ldots, 8$ in Excel. The required normal distributions $\Delta w = \mathrm{N}(0, 1)$ were generated by applying norm.inv(rand()), the inverse of the cumulative normal distribution, to samples from a uniform distribution $\mathrm{U}(0, 1)$. 50,000 replicates were performed for each combination of parameters, except for $\lambda = 0.8$ where 250,000 replicates were performed. We computed the probability of "falling off the cliff" (hitting the absorbing barrier) for $t \leq 40$. *Results:* the probability of falling off the cliff decreases sharply with increasing distance to absorbing barrier and increases sharply with increasing spectral radius λ. In particular, the probability of falling off decreases faster than exponentially with distance to the cliff, as shown by the left face of the top graph. The probability of falling off also decreases faster than exponentially with a decreasing spectral radius (increasing stability of deterministic model), as shown in the bottom graph.

Ornstein-Uhlenbeck processes have been widely used to analyze financial dynamics. Widely cited papers include Barndorff-Nielsen and Shephard (2001) and Vasicek (1977), who relate Ornstein-Uhlenbeck processes to the classical Merton-Black-Scholes model, but see Haug and Taleb (2011) for a critique (practical heuristics versus theory). Generalized Ornstein-Uhlenbeck processes, that is processes

$$dx = -rxdt + \sigma d\eta \tag{27}$$

in which the noise term η is not Gaussian are also widely used in financial dynamics.[13]

2.6. Extensions

Sections 2.1–2.5 described the stability-complexity relationship for "random" linear systems (with and without added Gaussian noise) as prototype models for the dynamics of ecological and economic networks. Here we briefly describe extensions to this linear theory.

2.6.1 More General "Random" Networks

Many other types of random networks have been studied. A detailed discussion would add greatly to the length of this paper, so we shall simply briefly cite some key references. One major class of networks is called *small-world networks* (Watts and Strogatz 1998): "these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them 'small-world' networks, by analogy with the small-world phenomenon (Milgram 1967; Kochen 1989) (popularly known as six degrees of separation [Guare 1990])."[14]

Small-world networks include *scale-free networks*, where the degree distribution follows a power law, rather than the normal distribution in Erdős-Rényi networks (Amaral et al. 2000). Barabási-Albert graphs form another important class (Barabási and Albert 1999); see also Barabási (2009), more generally, his book on *Network Science* (Barabási 2016), and Havlin et al. (2012) and references therein.

[13] See Benth, Kallsen, and Meyer-Brandis (2007) and Masuda (2004).
[14] See Jackson and Rogers (2005) for applications to economics.

One also sees various forms of preferential attachment in evolving/growing networks: edges are added preferentially to vertices of high degree (Newman 2001; Jeong, Néda, and Barabási 2003; Válesquez 2003). One can also weight the interactions in a random model (Wang and Chen 2008)—compare with our analysis of the world trade network in section 5.

Finally, one can also consider constrained random networks; for example, real food webs are not random (Pimm 1979, 1980; Pimm and Lawton 1980); in some cases this contributes to their stability (Allesina and Pascual 2016), in others it reduces Lyapunov stability (Hastings, Juhasz, and Schreiber 1992). Despite this nonrandomness, random matrix models can also serve as useful *neutral models*, providing benchmarks for stability analysis so that one can assess the effects of structure (nonrandomness) upon stability.

2.6.2 Modular Networks

Finally, many networks are modular, and their modular structure appears to yield enhanced stability (Gillarranz 2017). One key example is the US power grid:

> - Electricity generated at power plants moves through a complex network of electricity substations, power lines, and distribution transformers before it reaches customers. In the United States, the power system consists of more than 7,300 power plants, nearly 160,000 miles of high-voltage power lines, and millions of low-voltage power lines and distribution transformers, which connect 145 million customers. […] "Local electricity grids are interconnected to form larger networks for reliability and commercial purposes. At the highest level, the United States power system in the Lower 48 states is made up of three main interconnections, which operate largely independently from each other with limited transfers of power between them.
> - The Eastern Interconnection encompasses the area east of the Rocky Mountains and a portion of northern Texas. The Eastern Interconnection consists of 36 balancing authorities: 31 in the United States and 5 in Canada.
> - The Western Interconnection encompasses the area from the Rockies west and consists of 37 balancing authorities: 34 in the United States, 2 in Canada, and 1 in Mexico.
> - The Electric Reliability Council of Texas (ERCOT) covers most, but not all, of Texas and consists of a single balancing authority.
>
> The network structure of the interconnections helps maintain the reliability of the power system by providing *multiple routes for power to flow* and by allowing generators to supply electricity to many load centers. *This redundancy helps prevent transmission line or power plant failures from causing interruptions in service.*

(from US Energy Information Administration 2016; emphasis added); redundancy due to multiple [parallel] routes for power is reminiscent of MacArthur's (1955) "multiple energy pathways" scenario.

2.6.3 Cascading Failures

Networks can provide a substrate for localized failures to rapidly propagate through a network. These are known as cascading failures (c.f. Crucitti, Latora, and Marchiori 2004).[15] The cascading failure of the 2003 blackout demonstrated the need to balance the underlying dynamics of the power grid with appropriate regulation; the rapidity of the failure emphasized the need to consider and protect against such failures in advance.

2.6.4 Robustness

Hines, Balasubramaniam, and Sanchez (2009) discussed how decentralized (modular) organization and "reciprocal altruism" promote survivability from "cascading failures in power grids." Weighted networks (Wang and Chen 2008) and small-world structures (Xia, Fan, and Hill 2010) also promote robustness in the face of cascading failures; on the other hand, "dead ends" can undermine stability of the power grid (Menck 2014).

2.6.6 Universality of the May-Wigner Transition

(from stability if $\alpha^2 nC < 1$ to instability if $\alpha^2 nC > 1$). This transition has also been observed in nonlinear dynamical systems (Sinha and Sinha 2005; Fyodorov and Khoruzhenko 2016; Ipsen 2017) in addition to the linear systems near equilibrium considered above. In particular, Sinha and Sinha (2005) consider: "the persistence of individual nodes in a network of randomly coupled nonlinear maps undergoing a wide range of local dynamics. We observe that the results of the May-Wigner theorem seem to be valid universally, namely, increasing the number of interactions per node or increasing interaction strength will give rise to increased likelihood of extinction."

Fyodorov and Khoruzhenko (2016) provide a more detailed description of the transition to instability. Ornstein-Uhlenbeck dynamics (see section 2.5) might also be extended to nonlinear systems (c.f Beale 1989).

Question: How far can universality be extended?

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[15] For cascading failures in the US grid, such as the failures that caused the 2003 blackout, see Lerner (2013).

2.6.7 Feasibility

The May-Wigner stability theorem concerned linear stability near a steady state. In considering nonlinear systems in ecology and economics, we need to explore feasibility of steady states as well as their stability. Here a steady state is called "feasible" if all state variables are positive: One cannot have negative populations! An economic entity with negative net worth is fundamentally different from one with positive net worth. Stone (2018) and Dougoud et al. (2018) discuss the importance of feasibility in ecosystem modeling.

2.6.8 Evolution/Assembly/Design to the Edge of Stability

"The economy [can be considered] as an evolving complex system" (Blume and Durlauf 2005; see also Arthur, Durlauf, and Lane [2018] and Anderson, Arrow, and Pines [2018]), in which one can see emergent dynamics, that is system-level dynamics that cannot be easily derived from an examination of dynamics at the component level (c.f. Sinha and Sinha 2006). We posit *evolution/assembly/design to the edge of stability* in complex financial systems, as financial institutions seek out profits. Compare the formation of complex yet fragile food webs in tropical environments described in section 2.1 above, as well as the concept of evolution to the edge of chaos in many complex systems (Lewin 1999; Packard 1988; Kauffman and Johnse 1991), and in particular economic systems (Oxley and George 2007; Beinhocker 1997) in which linearized stability analysis can readily fail (Oxley and George 2007).

Bak et al. (1992) described this evolution in economics as *self-organized criticality*, dynamics at the limit of stability in which one may see "avalanches" of all scales whose magnitudes follow a power law distribution (compare the Gutenberg-Richter [1944] law for earthquakes). Power law distributions appear in the Gutenberg-Richter law as evidence of self-organized criticality (Bak and Tang 1989; Bak et al. 2002). Power law distributions are also one example of fat-tailed distributions, yielding fragility and "Black Swan" (Taleb 2007, 2010) events. Fluctuations in self-organized criticality are one example of *fractal dynamics* (Mandelbrot 1982; Hastings and Sugihara 1993). Samuelson's observation that "properly anticipated prices fluctuate randomly" (Samuleson 1965; Merton 2006) can be considered the antecedent to these dynamics in that a random walk is perhaps the prototype random fractal, much as the Merton-Black-Scholes formula (Schachermayer and Teichmann 2008; Duffie 1998) is an application of Ornstein-Uhlenbeck dynamics.

3. WHAT THIS MEANS FOR ECONOMICS

We briefly raise some questions implicit in May, Levin, and Sugihara's (2008) "Ecology for Bankers," including the effects of structure upon stability and the potential for forecasting.

3.1 The Nature of Stability

Suppose one observes low variability in a complex system (thus high stability by one measure; see tables 1 and 2 in sections 2.1 and 2.4, respectively. For complex stochastic systems modelled by [discrete-time] Ornstein-Uhlenbeck processes [equation 20])

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t) + \sigma\Delta\boldsymbol{w}(t),$$

or nonlinear generalizations, variability as measured by the standard deviation of $\{\boldsymbol{x}(t)\}$ at large times depends both upon the Lyapunov stability of the corresponding deterministic system

$$\boldsymbol{x}(t+1) = A\boldsymbol{x}(t)$$

and the environment—the amount σ of "exogenous" noise. Namely, the standard deviation of $\{\boldsymbol{x}(t)\}$ at large times is given by the ratio

$$\sigma/\sqrt{1-\lambda^2} \tag{28}$$

where λ, the size of the dominant eigenvalue of A, describes the Lyapunov stability of the underlying deterministic system. Thus a system evolving in a low-noise environment (compare Minsky's growth phase [Dymski 2009; Minsky 1975, 1986,1993]) might have low variability, only to fail if the noise level increases, increasing the standard deviation of $\{\boldsymbol{x}(t)\}$, or the distance to a fiscal cliff (see section 2.5) decreases (increasing fragility in a manner analogous to Minsky's Ponzi phase).

3.2 The Substrate for Failure: Too Big? Too Centralized? Too Correlated? Too Much Leverage/Not Enough Reserves?

As Heiberger (2014) observed: "Despite many efforts crises on financial markets are in large part still scientific black-boxes. In this paper, we use a winner-take-all approach to construct a longitudinal network of S&P 500 companies and their correlations between 2000 and 2012. A comparison to complex ecosystems is drawn, especially whether the May-Wigner theorem can describe real-world economic phenomena. The results confirm the utility of the May-Wigner theorem as a stability indicator for the US stock market, since its development matches with the two major crises of this period, the dot-com bubble and, particularly, the financial crisis. In those times of financial turmoil, *the stock network changes its composition, but unlike ecological systems it tightens and the disassortative structure of prosperous markets transforms into a more centralized topology.*" (emphasis added)[16]

The answer to the first question, likely "all of the above" could serve as a call for more effective forecasting (Sidorowich and Farmer [2018] ask "Can New Approaches to Nonlinear Modeling Improve Economic Forecasts?"; see also Aymanns et al.'s [2018] "Models of financial stability and their application" and Arrow et al.'s [2018] discussion of economic cycles), leading to more appropriate regulation. Empirical dynamical modeling, which presupposes no underlying model, may also be useful (Ye et al. 2015).

3.2.1 Remarks. See, e.g., Hastings, Young-Taft, and Wang (2019) and references therein for the use of interest rate spreads and dynamics for forecasting recessions in US economic cycles.

We now consider how the structure of the world trade network—dominated by a few to 10–30 large economies—limits the ability to forecast stability.

[16] See also Bardoscia et al. (2017) for pathways towards instability, discussed in the Introduction, Junior and Franca (2012) for correlation in times of crisis; and León and Berndsen (2014) for "challenges arising from financial networks' modular scale-free [and thus small-world] architecture."

4. IT'S A SMALL WORLD: RANDOM MATRIX MODELS FOR THE WORLD TRADE NETWORK

"The network formed by the trade relationships between all world countries, or *World Trade Web* (WTW). Each (directed) link is weighted by the amount of wealth flowing between two countries, and each country is characterized by the value of its *Gross Domestic Product* (GDP)." (Garlaschelli et al. 2007) May-Wigner is asymptotic result, we shall see that the WTW is effectively small.

4.1 Scaling in the World Trade Web

In previous sections we considered the effect of how interaction strength scales with vertex degree assuming i.i.d. random interactions. However, interactions in many real economic networks, such as the trade and financial networks, vary widely in scale.

If we consider the world trade network, trade between two countries is closely approximated by the well-known "gravity law" (Tinbergen 1962; Pöyhönen 1963; Krugman 1980); also the recent review (Garlaschelli et al. 2007) and references therein, states that trade between two countries is proportional to the product of their GDPs divided by the "difficulty of trade between them," the latter frequently expressed as a distance.

$$\text{trade between country } i \text{ and country } j = const \times \frac{\text{GDP}_i \times \text{GDP}_j}{\text{dist}(i,j)} \qquad (24)$$

With recent sharp declines in shipping costs, we shall consider a simplified case in which $\text{dist}(i,j) = 1$, resulting in the following simplified formula.

$$\text{trade between country } i \text{ and country } j = const \times \text{GDP}_i \times \text{GDP}_j \qquad (25)$$

Moreover, the GDP scales as a power law at the high end (Garlaschelli et al. 2007, Fagiolo et al. 2009) (figure 4), and this behavior may be universal (Garlaschelli et al. 2007; Montroll and Shlesinger 1982; Solomon and Richmond 2001, 2002; Reed and Hughes 2002; Mitzenmacher 2004); that is, the GDP of the n^{th} largest economy scales as

$$\mathrm{GDP}_n \sim n^{-b} \tag{26}$$

for some scaling (Zipf) exponent b. We found $b \approx 1$, thus

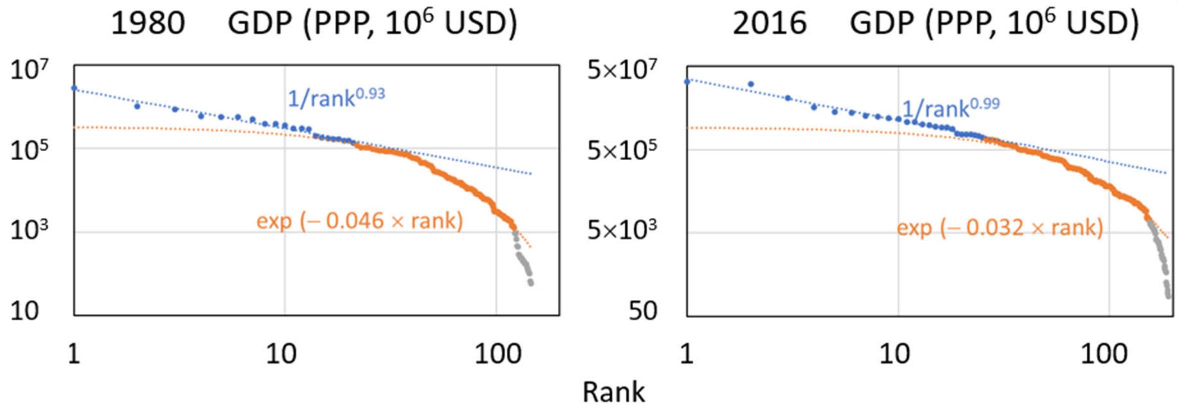
$$\mathrm{GDP}_n \sim 1/n \tag{27}$$

Power laws are sometimes referred to as Pareto distributions or in the case $b = 1$, Zipf distributions (Adamic n.d.).[17]

Garlaschelli et al. (2007) had argued that "the dynamics of all GDP values and the evolution of the WTW (trade flow and topology) are tightly coupled. The probability that two countries are connected [with weight above a given threshold] depends on their GDP values… On the other hand, the topology is shown to determine the GDP values due to the exchange between countries." In comparison, Chaney (2018) showed how the denominator (distance) in gravity law arises from a Pareto/power law distribution of sizes of firms.

[17] The use of methods from physics in economics is known as econophysics (Yakovenko 2009; Yakovenko and Rosser 2009). See Clauset, Shalizi, and Newman (2009) for a caution on fitting power laws.

Figure 4. GDP and per capita GDP Follow Power Laws at High End



Notes: The data appears to follow a Zipf power law scaling for the largest GDPs (GDP ~ 1/rank), followed by a middle region with exponential fall off, and finally the lowest values fall off like the tail of a normal distribution (exponential with the square of rank). There are suggestions that the data is not lognormal. The Shapiro-Wilks test for normality of log-transformed GDP's yields $p = 0.0448$ for 1980 data and $p = 0.1505$ for 2016 data, but these data sets are not independent (a linear analysis of log-transformed data yields $R^2 = 0.718$, N = 139, $p < 0.0001$) (Vassarstats, http://vassarstats.net/corr_big.html, May 23, 2019). Further analysis with Vassarstats is shown in table 2.

Table 2. Statistical Analysis of GDP Data

	Power law region (blue)	**Exponential fall-off region (orange)**
1980 GDP	GDP ~ $\text{rank}^{-0.93}$ $R^2 = 0.969$ N = 21, $p < 0.0001$ Lower bound: 1.2×10^{11} USD	GDP ~ exp (– 0.046 × rank) $R^2 = 0.996$ N = 101, $p < 0.0001$ Lower bound: 10^9 USD
2016 GDP	GDP ~ $\text{rank}^{-0.99}$ $R^2 = 0.978$ N = 25, $p < 0.0001$ Lower bound: 9×10^{11} USD	GDP ~ exp (– 0.032 × rank) $R^2 = 0.991$ N = 129, $p < 0.0001$ Lower bound: 10^{10} USD

Methodology: Scaling regions were estimated with the goal of maximizing the coefficient of correlation. The transition points are thus approximate. Small changes in the transition points have little effect upon the slopes, parameters and R^2.

Source: Data from Wikipedia (2020), IMF (2018): "This is an alphabetical list of countries by past and projected Gross Domestic Product, based on the Purchasing Power Parity (PPP) methodology, not on market exchange rates. Values are given in USDs. These notional figures have been taken from the International Monetary Fund's World Economic Outlook (WEO) Database, October 2018 Edition."

4.2. A Simplified Model: Scaled Random Matrices

We investigate the following simplified model for the stability of random systems scaled like the WTW. The model combines random matrix theory, the observed empirical power law for GDPs, and the gravity model with denominator 1 for the amount of trade between two countries.

Let A be an $n \times n$ random matrix whose entries A_{ij} are chosen independently from scaled uniform distributions

$$A_{ij} = {}^{1}/_{(ij)^{rb}}\ U(-0.5, 0.5) \tag{28}$$

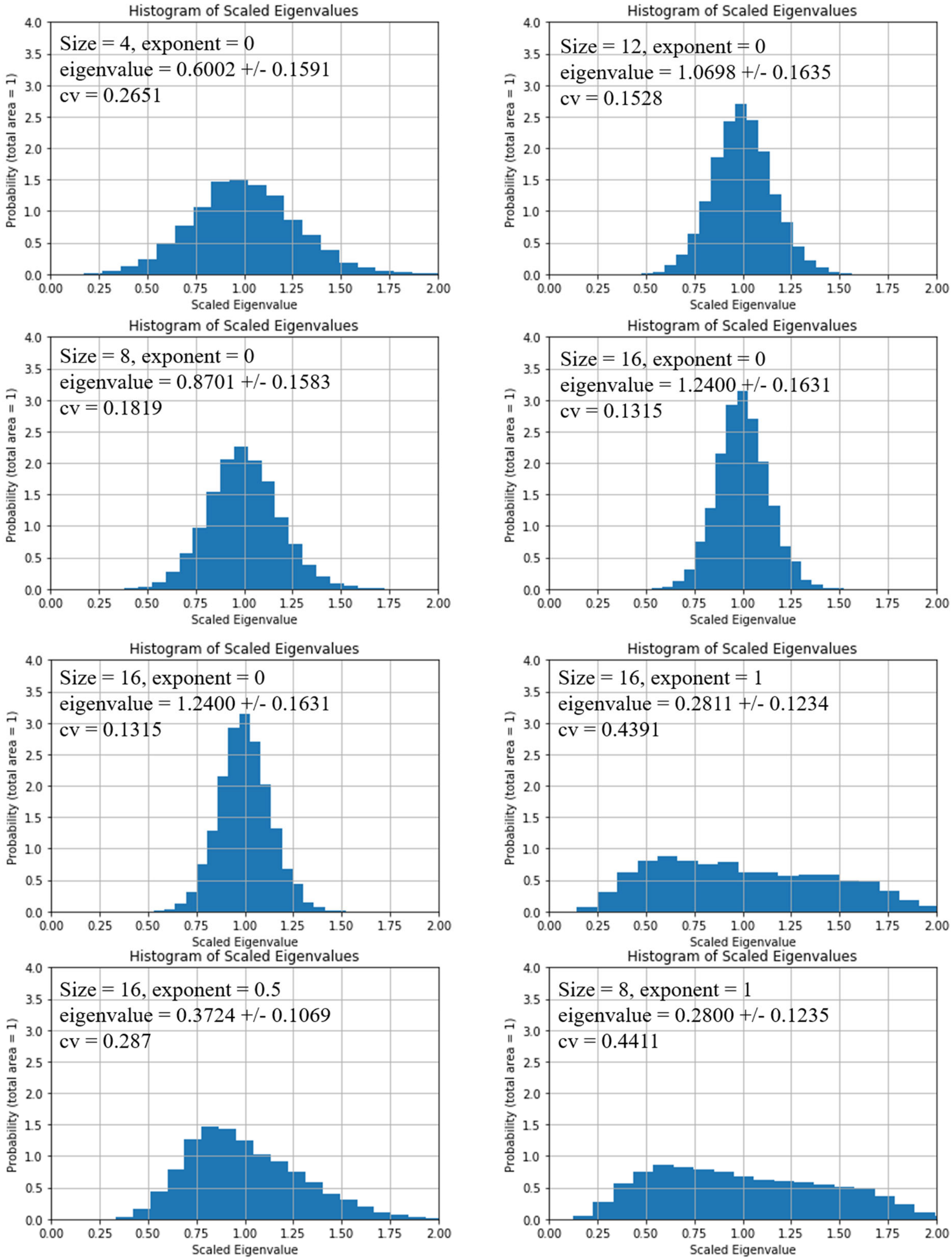
Here n denotes the number of countries in the model WTW, r measures how interaction strength scales with the amount of trade, b denotes the Zipf exponent in the power law distribution of world GDPs by country, and $U(-0.5, 0.5)$ denotes a uniformly distributed random variable taking values in the interval $(-0.5, 0.5)$.

Since our interest here is the spread of eigenvalues and its effect on predictability, we did not introduce any other scaling in interaction strengths. Instead, we rescaled the spectral radius (size of dominant eigenvalue) to 1, which can be accomplished by applying a similar scaling factor to all interaction strengths, thus not affecting relative interaction strengths.

4.2.1 Methods

We found the spectral radius with the power law in Python, ran 10,000 replicates for each set of parameters, and reported the mean, standard deviation, and histogram. Our results are shown in figure 5, where we scaled the resulting eigenvalues instead of scaling the interaction strength.

Figure 5. Distribution of Dominant Eigenvalues in Simulations of Simple Model "Trade" Networks



4.2.2 Conclusions

As shown in figure 5, small, economically relevant random matrices display a wide spread in dominant eigenvalues, thus May's criterion

$$\alpha^2 nC < 1$$

may not provide a sufficiently sharp bound on the size of the dominant eigenvalue to adequately insure stability. For example, for small (8×8) systems with interactions scaled by a combination of the gravity law for trade and the empirical distribution of the sizes of the largest economies, the coefficient of variability of the dominant eigenvalue is 0.44, with a range up to twice its average value. More generally, in many ways, the world economy is "effectively small," limiting the utility of asymptotic results.

Thus, it may be difficult to regulate or control the evolution/design/assembly to the limits of forecast stability.

5. DISCUSSION

We have aimed to provide a quasihistorical overview of the key mathematical ideas underlying important common features of ecological and economic networks. The world economic system has grown increasingly complex and interconnected in many ways:

> (1) Real and financial corporations operate largely independently of national boundaries (Chowla and Bolton 2005; Oxfamblogs 2011). In fact, by 2015, 69 of the largest 100 economic entities were businesses: "the US, China, Germany, Japan, France and the UK make up the top six economic entities followed by Italy, Brazil and Canada. Walmart ranks as the 10th largest, followed by China's electricity monopoly State Grid at number 14, China National Petroleum at 15 and Chinese oil firm Sinopec Group at 16. Apple ranked 26th behind the 18th-

placed Royal Dutch Shell, with Exxon Mobil at 21, Volkswagen at 22 and Toyota at 23" (Lordon 1997).

(2) World trade has become larger and more complex, with extensive trade in intermediate goods (global supply chains).

(3) Finance has become an international electronic-networked phenomenon.

(4) A variety of central bank, fiscal, and regulatory activity drives trajectories in interacting ways.

(5) Finally, the current econo-system involves a broad range of time scales (Lordon 1997), from the millisecond scale of high-frequency trading (recall the Flash Crash of May 6, 2010) (Kirilenko et al. 2017) to the 10+ year scale of major infrastructure structure projects.

These changes challenge our ability to ability to understand, manage, and forecast economic trajectories in a complex, high-dimensional state space, in the absence of stationarity and at best limited applicability of ergodicity.

One might hope and expect that the theory and applications of random matrices would provide a framework for exploring such complex dynamics. The theory of random matrices has played important roles in physics beginning with Wigner (1955, 1958, 1967) (28,000 papers listed in Google Scholar, June 18, 2019), ecology following May (1972) (2,200 papers listed in Google Scholar, June 18, 2019), and economics (c.f. "Ecology for bankers" [May, Levin, and Sugihara 2008], 4,800 papers listed in Google Scholar, June 18, 2019). Hopefully extensions of the May-Wigner stability theorem discussed above will lead to improved understanding and control of the complex "econo-system" as well as mitigation of the effects of dangerous interactions within the system.

5.1. Future Work

We are planning to extend our work on the dynamics of economic systems and also to examine the role of modularity and analogs of circuit breakers in power grids to prevent cascading failure in other networks. Here are some specific goals in understanding the dynamics of economic systems.

(1) Understand/incorporate/analyze multilevel economic networks including the major players, countries, banks, and firms, within an extended May-Wigner framework.
(2) Incorporate multiple time scales within the model.
(3) Develop circuit breakers to better isolate potential shocks to the economic system and forestall cascading failures.
(4) Improve forecasting, or at least better understand the limits of forecasting.

One can also consider the spread of COVID-19 as a cascading event (failure), in which case network science may provide useful ideas about curtailing its growth and spread, and ultimately may help design a more resilient, less fragile city of the future.

5.2. Limitations

We have not discussed the key role of nonlinearity beyond a few citations and we have not answered (or even attempted to answer) how we can best prevent or at least mitigate future financial crises.

We conclude with two key observations, the first due to Battiston et al. (2016):

> Traditional economic theory could not explain, much less predict, the near collapse of the financial system and its long-lasting effects on the global economy. Since the 2008 crisis, there has been increasing interest in using ideas from complexity theory to make sense of economic and financial markets. Concepts, such as tipping points, networks, contagion, feedback, and resilience have entered the financial and regulatory lexicon, but actual use of complexity models and results remains at an early stage. Recent insights and techniques offer potential for better monitoring and management of highly interconnected economic and financial systems and, thus, may help anticipate and manage future crises.

Secondly, translating Donohue et al. (2016) from ecology to economics in the spirit of “Ecology for Bankers” (May, Levin, and Sugihara 2008), we need more conversations among theorists, empiricists, and regulators.

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